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FACILITATING STUDENTS' GEOMETRIC THINKING THROUGH VAN HIELE'S PHASE-BASED LEARNING USING TANGRAM

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ABSTRACT

The aim of this study was to determine the effects of Van Hiele's phases of learning using tangrams on 3rd grade primary school students' levels of geometric thinking at the first (visual) and second (analysis) level. The study further investigated if high, moderate and low ability students acquire better mastery in geometric thinking at the end of tangram activities. Pre-test and post-test single group experimental design was employed in the study. A total of 221 students enrolled in Grade Three during the 2013 educational year formed the sample. The students learned Two-dimensional geometry and Symmetry through the Van Hiele's phases of learning using tangram. A geometric thinking test was administered to students before and after the intervention. The intervention took place for 3 hours. Paired samples t-tests comparing the mean scores of geometric thinking pre-test and the post-test were computed to determine if a significant difference existed. One-way Multivariate Analysis of Variances (MANOVA) was conducted to compare the students' pretest and posttest mean scores across the three groups: high, moderate and low ability students. The results found that there were significant differences between pre-test and post-test in students' geometric thinking. It was also found that Van Hiele's phases of learning using tangrams was able to significantly promote geometric thinking in the van Hiele's first (visual) and second (analysis) level among high, moderate and low ability students. Low ability students were observed to have the greatest improvement score compared to moderate and high ability students. Thus, the Van Hiele's phases of learning using tangram can be applied in primary school mathematics to help students achieve better level of geometric thinking.

Keywords: Geometric Thinking, Van Hiele's Phases of Learning, Primary School Students, Tangram

1. INTRODUCTION

Learning geometry for elementary learners relies on their level of thinking (Van Hiele, 1999; Clement and Sarama, 2000; Ho, 2003; Dindyal, 2007). Past research had indicated that young students from different level of thinking perceive geometric shapes differently (Clement and Sarama, 2000; Ho, 2003; Wu and Ma, 2006). In fact, their perception towards world (e.g., shapes) is quite dissimilar to the adults (Piaget, 1929). For elementary learners, they first grasp the idea of geometric shapes by visualization (Costa *et al*., 2009) and this is done by recognizing the shapes by their physical appearances based on their real life experiences (Wu and Ma, 2006; Ozerem, 2012).

Over the years, young students are frequently found to have numerous misconceptions in geometry (Ozerem, 2012). Mack (2007) claimed that most of the students could speak out the mathematical names for square, triangle, rectangle and circle, but sometimes they are perplexed when the shapes are rotated. This is due to a mismatch between formal definition and their mental

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images of geometric shapes (Archavsky and Goldenberg, 2005; Mack, 2007). Even for those who have strong development in conceptions, they could also be inconsistent with the learned Mathematics concept (Ozerem, 2012). Some researchers had attempted to study student's abilities in understanding basic shapes (e.g., circle, triangle and quadrilateral). They found many students to have problem in identifying quadrilateral, followed by triangle and then circle (Clement and Sarama, 2000; Wu and Ma, 2006). Young students, thus, need to develop and build up the right schemata about two-dimensional geometric shapes and their properties before they continue their geometry lessons in the upper education level. Teachers should provide learning experience that match with children's level of thinking about geometric shapes.

1.1. Background

1.1.1. Geometric Thinking Skill

Van Hiele (1986) proposes a five-level model describing how children learn geometry. These levels are product of experience and instruction, moving from visualisation, analysis, informal deduction and deduction to rigour. According to the first ('visual') level of Van Hiele (1986)'s geometric model of thinking, learners visually recognize shapes and figures by their global appearance. For example, learners recognize triangles, squares, parallelograms and so forth by their shape, but they do not explicitly identify the properties of these figures.

At the second ('analysis') level, learners start analyzing the properties of figures and learn the appropriate technical terminology for describing them, but they do not interrelate figures or properties of figures (for example, a rhombus is a figure with four equal sides). At the third ('informal deduction') level, learners can identify relationship between classes of figures and discover properties of classes of figures by simple logical deduction (for example, a square is considered a rectangle because it has all the properties of a rectangle). Spear (1993) postulates that the first three levels are within the capacity of primary school learners. Thus, learners at the lower primary school level should at least attain the first two van Hiele's levels in order to move effectively from one level of thinking to another.

1.2. Manipulative Teaching and Learning Aids

The van Hiele's theory stresses on the use of hands-on manipulatives in teaching geometry to facilitate the transition from one level of thinking to the next (Fuys *et al*., 1988). Research on the teaching and learning of geometry also indicates that physical experience, especially the physical manipulation of geometric shapes, are necessary in order for students to gain a firm understanding of geometric relationships and that manipulative teaching and learning aids have much to offer (Tchoshanov, 2011). Manipulative teaching and learning aids are physical objects that can be touched, turned, rearranged and collected (Brown, 2007). In other words, manipulative aids are physical objects that appeal to several of the senses where students are able to see, touch, handle and move. Manipulatives help children in bridging their concrete sensory environment to the abstract understanding of Mathematics (Bayram, 2004; Trespalacios, 2008; Ojose and Sexton, 2009). Battista and Clements (1988) argue that geometry at the primary school level should be 'the study of objects, motions and relationships in a spatial environment'. This means that primary school students' first experience with geometry should give emphasis to the informal study of physical shapes and their properties and have as their major goal the development of students' intuition and knowledge about their spatial environment. Subsequent experiences should involve students in analyzing geometric concepts and relationships in increasingly formal settings.

1.3. Tangrams

Singh (2004) asserts that tangrams are stimulating manipulative learning and teaching aids that help young students to acquire geometry thinking and reasoning process. A tangram is the oldest Chinese puzzle that consists of seven geometric pieces of shapes, called *tans* (Tian, 2012). The seven pieces include a square, a parallelogram, two big right triangles, a medium sized right triangle and two small right triangles. The three basic shapes consist of a triangle, a square and a parallelogram, which fit together in various ways to form polygons such as a large square, rectangle, or triangle. Also, these *tans* can be arranged in a variety of figures such as birds and animals (Tian, 2012).

A recent study has found that tangrams are useful manipulative aids in developing the concept of geometry (Lin *et al*., 2011). Tangrams allow children to develop geometric concepts by categorising, comparing and working out the puzzle and thereupon to solve problems in geometric contexts. When children touch and manipulate concrete objects, they become more proficient in knowing positions or locations in space (for examples: Above, horizontal) and structure (for example: number of parallel sides). Ultimately, hands-on investigation of geometric objects helps young children develop a strong intuitive grasp of geometric properties and relationships (NJMCF, 1995). Studies show that tangrams inspire children's observation, imagination, shape analysis, creativity and logical thinking (Olkun *et al*., 2005; Yang and Chen, 2010). Accordingly, learning geometry with tangrams can help children to develop their skills of geometry vocabulary, shape identification, shape orientation and discover relationships between and among the 2-dimensional geometric shapes (Bohning and Althouse, 1997; NCTM, 2003).

1.4. Difficulties in Learning Geometry among Elementary Learners

According to Idris (2007), difficulties in learning geometry among elementary learners could be explained in terms of individual's cognitive development, instructional practices and materials and the mathematical system. Individual with better visual perception has an advantage in geometric reasoning (Walker *et al*., 2011). Individual cognitive ability is not just about visual perception, but also decision making, which is crucial to achieve higher-order thinking in learning geometry. Taiwanese scholar, Wu and Ma (2006) tried to examine young students' perception about triangle and quadrilateral based on van Hiele's level of thinking. Findings found that 1st to 4th graders were able to attain the visual level of thinking, while only some of the 3rd and 4th graders were able to identify geometric shapes by defining or describing them, which is the second level of van Hiele's geometric thinking.

Particularly, Clement and Sarama (2000) found that 3 to 6 years old students perceived triangle by relying on the "top point" of the shape. In addition, there must be a horizontal line as the base (Kaur, 2012). Thus, they could misjudge any shape or any triangular form which had curve sides as a form of triangle. As for the quadrilateral, students perceived that any long shape with four sides is a rectangle, such as parallelogram or trapezoids (Clement and Sarama, 2000). Gal and Lew (2008) provided evidence that the 9th grade low achievers in Korea had difficulties in classifying the 'special' parallelogram. This is because understanding of quadrilateral requires inclusion relation to classifying shapes, such as rectangle, square and rhombus, which are also a 'special' parallelogram (Gal and Lew, 2008).

Other than that, past researchers strongly emphasized that concrete experiences ought to be exposed at the primary level in bridging abstract concepts with the concrete objects (Zanzali, 2000; Kamina and Iyer, 2009), such as playing with models, drawing and sorting. Instead of the customary memorizing of mathematical name and attributes of 2-D shapes, the visual-assisted tools would help students to enhance thinking ability and make conclusion correctly as it is functioned as a mental reference (Gal and Lew, 2008; Abdullah and Zakaria, 2012; Keuroghlian, 2013). If students learn geometry by solely memorizing the definitions, they would not be able to perform in higher level task and thus, they may simply make decision incorrectly based on their own prototypes (Gal and Lew, 2008).

As for Malaysia, lower primary school curriculum was designed to enable the students to state the mathematical terms for the shapes and identify the properties for square, rectangles, triangles, cuboids, cylinders, spheres, cones and pyramids (Zanzali, 2000). Teaching Mathematics in Malaysian school classrooms is often reported as too teacher-centred (Idris, 2007). Such practice could eventually obstruct students' learning of geometry. As Van Hiele stated, "The transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program" (1986). Teachers hold the key to this transition from one level to the next. Researchers have pointed out that students' level of geometric thinking depends on how the instruction is delivered to them (Alex and Mammen, 2012; Abidin, 2013). If students become passive in learning geometry, a gap will eventually emerge between their thinking levels and the expected geometry learning outcomes (Gal and Lew, 2008; Kaur, 2012).

Some misconceptions in learning geometry could also be due to terminology and language (Lee and Ginsburg, 2009; Keuroghlian, 2013). For example, students' were puzzled by the word 'right angle' as the angle opens to the right, therefore, there is a 'left angle' if it is opened to the left (Mack, 2007). Lee and Ginsburg (2009) also found that the usage of language sometimes create misconception in learning among young students. For example, they found that misconception on terminology used for naming numbers occur in English medium classrooms, but not in a Korean medium classroom. It is essential that students need help to uncover these misconceptions and thus, build on correct perceptions.

It is plausible that students are incapable of understanding geometry due to a mismatch between their level of Van Hiele (1986) geometric thinking and level of instruction. On the other hand, it is also plausible that manipulative learning and teaching aids, particularly those of a concrete, hands-on nature, may have much to offer students who cannot comprehend abstract geometric concepts. According to the van Hieles, a student progresses through each level of thought as a result of instruction that is organized into five phases of learning. The five phases are inquiry, guided orientation, explicitation, free orientation and integration. Thus, manipulative learning and teaching aids such as tangrams may be a useful tool to help young students visualise and analyze geometric shapes, while at the same time providing Van Hiele's phases of learning environment that promotes geometric thinking in them.

These arguments present an interesting conundrum. Will Van Hiele's phases of learning using tangrams as manipulative learning and teaching aids assists moderate and low ability students to develop their geometric thinking? Or is the use of tangram has more impact on high ability students in learning geometry? Through the findings of this research, it can give insights to mathematics educators on the role of tangrams and how the techniques and processes of using tangram in teaching geometrical concepts can make the abstract concept of 'geometry' comprehensible to moderate and low ability students in primary schools.

1.5. Purpose of the Study

There has been little research presented concerning the impact of Van Hiele's phases of learning using tangram on primary school student's geometric thinking based on their thinking level. This study was thus undertaken to find out the extent to which the use of tangram as an in-class activity using the Van Hiele's phases of learning could assist 3rd grade primary students of high, moderate and low ability in developing their levels of geometric thinking at the first (visual) and second (analysis) level. The phases involved were inquiry, guided orientation, explication, free orientation and integration. Tangram puzzle was used as a medium to support the learning environment using Van Hiele's five phases of learning.

This study focuses only on the first two level hierarchy of thinking processes of Van Hiele (1986)'s geometric model as they are the most pertinent ones for lower level of primary school geometry. It is hypothesized that children develop geometric thinking such as recognizing shapes and figures by their global appearance and naming additional geometric shapes which are constructed from the given tans (visualisation) and classifying of shapes according to properties and deriving generalisations inductively (analysis).

1.6. Research Questions

This study addresses the following research questions:

- Is there a significant difference between post-test and pre-test mean scores in geometric thinking among Grade 3 students at the end of Van Hiele's phases of learning using tangrams?
- Is there a significant difference between post and pre-test mean scores in geometric thinking among the (i) high, (ii) moderate and (iii) low ability Grade 3 students at the end of Van Hiele's phases of learning using tangrams?
- Is there a significant difference between post and pre-test mean scores at the first level (visual) of geometric thinking among the (i) high, (ii) moderate and (iii) low ability Grade 3 students at the end of Van Hiele's phases of learning using tangrams?
- Is there a significant difference between post and pre-test mean scores at the second level (analysis) of geometric thinking among the (i) high, (ii) moderate and (iii) low ability Grade 3 students at the end of Van Hiele's phases of learning using tangrams?
- What are the Grade 3 students' insights and experiences about using tangrams in learning geometry?

2. MATERIALS AND METHODS

2.1. Sample and Sampling Method

Purposive sampling technique was chosen to select schools that formed the sample of the study. Purposive sampling was used so as to minimise experimental contamination (Fraenkel and Wallen, 2000). The research sample was composed of 221 students who were enrolled in Year Three classes in the 2013 academic year in a Malaysian primary school. About 40 Grade Three primary school students with mix abilities were instructed in the same classroom. 6 classes were involved in this study. The students from the 6 selected classes studied the same topics and followed the same learning materials. All students were required to take a pre-test prior to the start of the intervention. After the completion of the intervention, students were re-evaluated with a post-test.

2.2. Research Design

A single group pre test-post test experimental research design was employed in the study. The single pre test-post test group design involves collecting information on the level of students prior to and following Van Hiele's phases of learning with tangram activities. This involves pre-testing and subsequent measurement in post-testing of students' geometric thinking after implementation. This design provides a significant improvement over the one-shot study because it measures change in the outcome indicators; whereby in this research, the outcome indicators are students' geometric thinking. At the same time, the perceptions on the use of tangram on facilitating teaching and learning were explored through the students' written feedback for the open questions which included:- "I like/do not like to learn geometry by using tangram because…". The topics chosen in the intervention were Two-Dimensional Shapes and Symmetry, two of the topics in Primary Year Three Mathematics curriculum syllabus.

The quantitative data was collected through the pre- and post-test scores and were then analyzed using SPSS for Windows (version 19.0). One-way Multivariate Analysis of Variances (MANOVA) was employed to analyze if there were significant differences between the post and pre test scores among high, moderate and low ability Grade 3 students. The qualitative data was collected through written reflection of the students. Specific themes and variables as proposed by Estrada (2007) were used as guidelines when analyzing the students' written reflection. The students' written reflections were translated from Malay language into the English version using "Back Translation Method".

2.3. Research Instrument

A geometric thinking test was designed according to visualization and analysis level of van Hieles' thinking model. It was used as the pre- and post-test. The test consisted of 30 multiple choice items measuring 2-D geometric shapes and symmetry posed in a pencil-and-paper format. The items were grouped into two categories of geometric thinking with 15 items each at visualization and analysis level. All the items presented were attached with geometric diagrams or a class of figures. Students needed to make shape identification, classification and generalization for the given geometric diagrams. The geometric thinking tests were validated by mathematics experts from the School of Mathematics Education Department. The reliability of the test was measured using a pioneer sample of 30 Grade Three learners who were randomly selected to go through the intervention and take the test. Kuder-Richardson inter-term reliability was found to have the score of 0.85.

2.4. Applying the van Hiele's 5 Phases of Learning in Tangram Activities

The Van Hiele (1986)'s five phases of learning was introduced to students throughout the geometry lesson. Obviously, these 5 phases progress from one level to the next involving: Inquiry, guided orientation, explication, free orientation and integration. The phases are described in the following tangram activities for supporting a transition from visualization level to analysis level.

2.5. Activities Prior to Intervention

Students were individually asked to cut a lined tangram square (17.5×17.5 cm) into 7 pieces. The 7 pieces were numbered on their topsides for reference in directions and discussions of the activities. After students have explored and familiarised themselves with the tangram pieces, students were taken to the inquiry phase level.

2.6. Inquiry Phase

Discovers certain structures by examining holistically examples and non-examples.

At this initial stage, students worked cooperatively in a group of 3-4. They were required to manipulate, construct and recognize geometric shapes by using a combination of tangram and concrete objects in their surroundings. For example, they observe 2D front view of the ruler, eraser, pen, bottle, food container lid and paper clips to describe the characteristics of polygons and non polygons. This activity leads students to get acquainted with the different geometric shapes.

This activity also leads students to notice that joining the tangram pieces sometimes make a shape that is not the same as one of the original pieces. For example two small right triangles will become a square. In solving puzzles like these, students work visually with angles that fit and sides that match.

2.7. Guided Orientation

To examine the properties of the geometric shapes.

At this stage, the learners explore the 2-dimensional shapes through carefully guided activities in order to record the properties of the shape. For example, while examining an equilateral triangle, students found that it has such property as three equal sides; three equal angles and three symmetries.

Other than the 7 tangram pieces, the surrounding natural objects such as leaves and flowers were also used and folded in order to produce the lines of symmetry. Students were asked to state the number of line symmetry in the object.

2.8. Explication

Introduces terminology for the properties and different types of polygons.

At this stage, teachers introduced new terms for describing the properties and different types of the geometric shape using accurate and appropriate language. For example:- congruent, corners, straight sides, right angles, face, equilateral triangle, square, quadrilateral, regular and irregular polygons, pentagon, hexagon, heptagon and octagon.

2.9. Free Orientation

Explores new geometric shapes

The students learnt by doing more complex tasks to find his/her own way in the network of relations. For example, by knowing properties of pentagon, students investigated these properties for a new shape, such as hexagon, heptagon and octagon.

2.10. "Integration"

Summarize the properties of a geometric shape.

At this stage, students began to build an overview of all that they have learned about a geometric shape. For example, students composed a rule that an octagon has eight equal sides; its corners are the same–all are equal angles; and it can be folded to exhibit 8 line symmetry. Students also learned about other polygons such as heptagon in a similar manner.

In order to achieve its potential as a manipulative learning and teaching tool, tangrams were accompanied by a series of exercises that included curricular scaffolding (e.g. questions, activities) to provide a structure to guide students through the van Hiele's five phases of learning process. The activities in this worksheet helped students to engage with geometric shapes. For example, with the diagrams provided in the worksheet, student (i) determine the lines of symmetry from different shapes and (ii) draw a dotted line on each shape to represent a line of symmetry.

Additionally, a lesson plan was specifically designed to be used by the teachers-to reduce teacher's variation in carrying out the intervention as much as possible.

3. RESULTS

For analyses, students were stratified into high, moderate and low achievers in geometric thinking according to their performance scores in the pre-test. The participants' level of geometric thinking was determined according to the successfully answered questions based on the following criteria:

- The learner was classified as a high achiever if he/she could answer 70% or more of the questions given correctly
- The learner was classified as a moderate achiever if he/she could answer 40%-69% of the questions given correctly
- The learner was classified as a low achiever if he/she could only answer 1%-39% of the questions given correctly

In this study, a paired-sample t-test was conducted to compare the mean scores of pre and post-test of geometric thinking among grade 3 students. The intervention is concluded as effective if the test showed a statistically significant result. However, the result from the paired sample t-test was insufficient in providing information to compare the mean difference among the students' in gaining scores based on the different abilities level. In order to gain extra information for comparison among students of high, moderate and low ability level and geometric thinking test scores, one-way MANOVA was then tested using SPSS for Windows (version 19.0). Alpha was set at 95% level of significance. The Pillai's Trace as the multivariate test statistic was used to evaluate the multivariate differences. Pillai's Trace criterion was considered as the most powerful and robust statistic against violations of assumptions (Leech *et al*., 2005; Hsu *et al*., 2010; Field, 2013).

3.1. Results from Quantitative Data

Table 1 reported that there was a significant difference between pre and post-test in overall visualization and analysis levels of geometric thinking ($p = 0.00$, $p = 0.00$ and $p = 0.00$ respectively). **Table 1** also showed that gain score of overall visualization and analysis levels of geometric thinking was increased by 15.07, 16.57 and 11.77 respectively. The results hence suggested that the intervention did really have an effect in promoting students' geometric thinking in overall visualization and analysis levels.

As shown in **Table 2**, the gain scores for overall geometric thinking test of high, moderate and low ability students were increased as they underwent Van Hiele's phases of learning using tangrams, by 8.99, 13.59 and 22.65 respectively; visual level of geometric thinking test was increased by 8.05, 16.17 and 25.00 respectively; and the analysis level of geometric thinking gain scores was increased by 2.97, 11.26 and 20.59 respectively. These results indicated that the applied intervention was most effective among low ability students, sequentially followed by moderate and high ability students.

Table 1. Paired Sample T-Test for overall, first level (visual) and second level (analysis) of geometric thinking

	Paired differences				
	Mean	Std. Deviation	t	df	Sig. (2-tailed)
Paired Samples T-Test					
Post-Pre Test	15.07	12.623	-17.74	220	0.000
Post-Pre Visual	16.57	17.590	-14.04	221	0.000
Post-Pre Analysis	11.77	14.529	-12.04	220	0.000

*Significant at p<0.05

Table 2. Descriptive statistics of overall, first level (visual) and second level (analysis) of geometric thinking test

Ability level	N	Test	Mean	SD	Mean Difference
High	63	Post-Test	82.16	6.52	8.99
		Pre-Test	73.17	2.02	
Moderate	90	Post-Test	62.81	12.64	13.59
		Pre-Test	49.22	7.52	
Low	68	Post-Test	51.47	12.84	22.65
		Pre-Test	28.82	6.06	
High	63	Post-Visual	77.32	14.14	8.05
		Pre-Visual	69.27	9.04	
Moderate	90	Post-Visual	58.16	15.63	16.17
		Pre-Visual	41.99	11.75	
Low	68	Post-Visual	49.26	16.56	25.00
		Pre-Visual	24.26	9.33	
High	63	Post-Analysis	79.56	7.87	2.97
		Pre-Analysis	76.59	4.49	
Moderate	90	Post-Analysis	66.88	13.78	11.26
		Pre-Analysis	55.62	10.81	
Low	68	Post-Analysis	53.40	14.10	20.59
		Pre-Analysis	32.81	11.09	

*Significant at p<0.05

As shown in **Table 3**, a statistically significant MANOVA effect was obtained for overall, Pillai's Trace = 0.91, F(4,436) = 90.89, p = 0.00, indicating a difference among students' ability level on a linear combination of the pre-test and post-test; for visual level, a statistically significant MANOVA effect was obtained, Pillai's Trace = 0.75, F(4,436) = 66.10, p = 0.00, indicating a difference among students' ability level on a linear combination of the pre-visual and post-visual; and for analysis level, a statistically significant MANOVA effect was obtained, Pillai's Trace = 0.76, F(4,436) = 67.16, p = 0.00, indicating a difference between post test and pre test mean difference at the analysis level of geometric thinking mean scores between students among high, moderate and low ability students.

Table 4 is a summary of post hoc pair-wise comparisons among students across the three ability groups of their geometric thinking mean scores before and after intervention. The result revealed that the mean difference between high and moderate, high and low, moderate and low ability students was reduced accordingly by 4.60, 13.66 and 9.06 from their overall gain score; at visual level of geometric thinking, the mean difference between high and moderate, high and low, moderate and low ability students was reduced accordingly by 8.12, 16.95 and 10.83; at analysis level of geometric thinking, the mean difference between high and moderate, high and low, moderate and low ability students was reduced accordingly by 8.27, 17.61 and 9.34. Therefore, result suggested that van-Hiele phase based using tangram could reduce gap across students with different ability level at the analysis level of geometric thinking.

3.2. Result from the Open-Ended Questions

The responses from open-ended questions were analysed to investigate the learners' insights and experiences in tangram activities as well as the process of learning. The validity of open-ended response was determined by agreement between a Mathematics lecturer as independent rater and the researcher. The responses are described as follows.

Table 3. Multivariate tests among ability level with overall, visual and analysis level geometric thinking test

Effect		Value	F	Hypothesis df	Error df	Sig.
Multivariate tests-pillai's trace						
Ability Level	Overall	0.91	90.89	4	436	0.00
	Visual	0.75	66.10	4	436	0.00
	Analysis	0.76	67.16	4	436	0.00

*: Computed using alpha = 0.05

Table 4. Summary of post hoc pair-wise comparisons between students across the three ability groups at the overall, visual and analysis level of geometric thinking test

Ability Level (i)	Ability Level (ii)	Test	Mean difference	Reduced in mean difference
High	Moderate	Post-Test	19.35*	4.60
		Pre-Test	23.95*	
High	Low	Post-Test	30.69*	13.66
		Pre-Test	44.35*	
Moderate	Low	Post-Test	11.34*	9.06
		Pre-Test	20.40*	
High	Moderate	Post-Visual	19.16*	8.12
		Pre-Visual	27.28*	
High	Low	Post-Visual	28.06*	16.95
		Pre-Visual	45.01*	
Moderate	Low	Post-Visual	8.90*	10.83
		Pre-Visual	17.73*	
High	Moderate	Post-Analysis	12.69*	8.27
		Pre-Analysis	20.96*	
High	Low	Post-Analysis	26.16*	17.61
		Pre-Analysis	43.77*	
Moderate	Low	Post-Analysis	13.47*	9.34
		Pre-Analysis	22.81*	

*: The mean difference is significant at the 0.05 level

Almost all the students felt that the tangram activities were playful and hence enjoyable. They liked playing with tangram pieces and enjoyed creating their own shapes using their imaginations. They responded that: - "I like playing with tangram pieces. It looks like a puzzle and origami". "I can make my own shape that I love"; and "I like to do cut and paste activities, Many shapes can be designed from a piece of tangram set"; and "I love making different kinds of shapes using my imagination".

Students can arrange the tangram pieces in several ways; hence they enjoy finding new ways in their creation. They responded that:-"I have many ways to move around the tangram pieces. I can arrange and rearrange the pieces to create new shapes".

Students also experienced the joy of expressing themselves openly. They pointed out that:- "I can create whatever I wish".

Students also felt that the tangram activities can improve their knowledge about 2-D geometric shapes. Some of the related responses were: - "I can learn many geometric shapes from a set of tangram", "I love learning many new shapes at a time;" and "Many geometric concepts can be learned from tangram activities", "It helps me to understand the differences between polygon and non-polygon better".

Students generally felt that the activities had helped to stimulate their thinking about geometric shapes. Their feedbacks were:-: "the tangram helps to activate my brain to think of shapes that I would wish to create", "The tangram activities had given my brain a boost to think of shapes that I am going to create".

Many students expect more tangram activities in Mathematics classes. They wrote, "I am hoping to see more tangram activities in the future mathematics lesson", "I am interested in the way the teacher teaches geometry using tangram", "Mathematics is my favourite subject now". I love learning mathematics using tangram.

They also felt that the 7 tans tangram were easy to operate and thus, can carry out a lot of interesting activities. Related responses were:- "it is very easy to operate to produce different forms of figures", "Tangram has many uses, I can do a lot of interesting activities".

On the whole, primary school students have showed positive perception towards the use of tangram in the learning and teaching of geometry.

4. DISCUSSION

After conducting an analysis on the test scores, it was found that students performed significantly better on the post-test mean scores compared to the pre-test mean scores in geometric thinking. The result of this study shows that the use of tangram as an in-class activity following the Van Hiele's five phases of learning has effectively helped grade 3 students in promoting their geometric thinking.

It was also found that Van Hiele's phases of learning using tangrams was able to promote geometric thinking at the van Hiele's first (visual) and second (analysis) level among high, moderate and low ability students. Nevertheless, the result also indicated different degree of effectiveness among the three different achievement groups of students. Low ability students were observed to have greatest improvement score compared to moderate and high ability students in overall, as well as for the first (visual) and second (analysis) level of geometric thinking.

These results connect with other research findings and highlight the importance of selecting appropriate manipulative teaching and learning aids for lower achievement groups during mathematics instruction. For example, research review conducted by Strom (2009) found that the low achieving children showed more academically successful when using physical manipulatives. Physical manipulatives gave this group of students a multi-sensory learning experience that allowed them to become more involved in the class activities and conversations. In the current study, it was also found that low achieving students were influenced more by the treatment using tangrams. These similar results may indicate that low achieving students benefit more from visual and physical models that scaffold their geometry learning and support their visualization and analysis skills in meaningful ways.

The results further support the finding of previous research that tangrams inspire learner's observation and shape analysis and identification (Bohning and Althouse, 1997; Olkun *et al*., 2005; Yang and Chen, 2010; NCTM, 2003). Students taught according to the Van Hiele' phases of learning with the help of tangram activities had to explore and discover certain geometric shapes by observation and record directly the properties of the geometric shapes. Hands-on activities and concrete experience such as touching, turning, rearranging and combining tangram pieces into one to form new shapes provide an advantage to facilitate students in enhancing their visualization and analysis skills.

During explication phase, learners learned new terms for describing the properties and different types of the geometric shape using correct terminology. The free orientations activities have increased their ability to explore and think about new geometric shapes such as hexagon, heptagon and octagon. By summarizing the properties of a geometric shape during integration phase, students have attained a new level of thought about geometric shapes. Consequently, by following Van Hiele' phases of learning, tangrams help children to foster the development of their geometric thinking at the visualization and analysis level.

The students' responses from open-ended questions further showed that they enjoyed learning geometry using the tangrams. Students began to see that learning geometry as an activity enables them to discover new things and unleash their creativity. Students also found that it is indeed easier for them to develop a deeper understanding of geometrical concepts with the help of tangram puzzles. Students also found that tangram activities do not only stimulate their geometric thinking, but also promote their interest and motivation towards learning geometry.

5. CONCLUSION

Incorporating tangram activities in Van Hiele's 5 phases of learning that involves hands-on and investigative approach help Grade Three students to enhance visualization and analysis skills. This study also suggests that tangram activities when integrated with Van Hiele's 5 phases of learning provide added benefit for students with lower ability. Using tangram as manipulative teaching and learning aids allows low achieving students to move easily from visualization to analysis level of geometric thinking.

Learning geometry using tangram was perceived by most students as enjoyable to unleash their thinking and creativity. This study indicates that effective learning takes place when students actively experience the objects of study in appropriate contexts. Instruction that include sequences of hands-on activities and concrete experience, beginning with an exploratory phase, gradually building geometric concepts and related terms and culminating summary activities help students obtain an overview of the whole geometry shape that has been explored. Consequently, the theoretical approaches concerned with the development of the geometric thinking of students are important areas of pedagogical concern and it should be internalized by the mathematics educators.

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