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Predicting the Movement of Dentistry Students using Markov Chain

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Abstract: Developing and increasing the effectiveness of any system (university system) depends on continuous evaluation and improvement using quantitative methods based on quantitative measurement and efficient mathematical models that can predict the future of the system, discover its improvement requirements and increase its effectiveness . This study is an application of the Stochastics Markov Process for students of the College of Dentistry at Ibb University. It was carried out by building a model for the movement (transition) of student's cohorts in the college during the years of study and predicting the average times spent by the student until obtaining a degree in his/her track, as well as predicting the number of students expected to graduate for the post-study period. The study found that the average time for obtaining a bachelor's degree (graduation) is ordinary for the total students, although the average times for male students are less than the average times for female students.

Keywords: Stochastic Process, Random Markov Process, Transition Matrix, Absorbing State, Transient State, Recurrent State

Introduction

Science is getting more sophisticated day by day and modern administrative systems are getting more and more complex due to the development of their operations. Thus, they need a performance evaluation which aims at measuring the system's effectiveness and reliability, its performance, its financial cost and the optimal evaluation of any system aiming at predicting the behavior of the system in the future in a quantitative way (German, 2000; Haverkort, 1998).

The optimal evaluation of any system is usually done using methods based on quantitative measurement and based on an efficient mathematical model, through which we can predict the future of the system and discover the requirements to reach its optimal state including random process systems. The stochastic process plays a major role in predicting the future based on the present situation and this is one of the effective mathematical systems to determine the future, especially in the systems of variable and random inputs.

Educational systems (universities as a model) face huge challenges. One of the most important and competitive challenges has become of a global nature and has caused a set of factors on educational systems globally, which is the tendency towards the three-semester system, academic accreditation, evaluation and international classification of universities, there are many elements of evaluation of the performance of universities.

The most important elements in evaluation the universities include the students and the availability of an attractive, stimulating and thrilling university environment and the extent to which university systems possess the attractive academic environment and exemplary guidance to embrace students in desirable disciplines that are compatible with the labour market.

Therefore, when the system seeks to develop its tools, it gets rid of all the obstacles that lead to an increase in educational wastage (dropout - stumbling) in order to preserve resources and reduce the financial cost the country may wastage in case of the continuation of this phenomenon.

When you look at the university educational system, especially the educational movement and the transition of cohorts (the movement of students in the different academic levels at the university), it becomes clear that the movement (transition) is not known in advance, but rather a random movement based on probabilistic laws and if the optimal mathematical model is determined to study this movement, it will be possible to locate the educational wastage and all its related details.

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The transition of students between the different educational stages takes place in a random movement that corresponds to stochastic process.

The research attempts to use a Markov process to model student movement between university stages to help reduce educational wastage and financial cost.

Our aim is analysis of the student flow movement in the different stages of the research sample by using types of stochastic processes (Markov processes). The process of student transition from one stage of study to another has Markov property, since the transition between the different stages depends on the situation in the present and not the past.

The research analyzes sample data and creates a mathematical model, which is a stochastic process with Markov property.

Theoretical Framework

Concept of Stochastic Process

Definition (1): Let T be a subset of $[0, \infty)$. A family of random variables $\{X_t: t \in T\}$, indexed by T, is called a stochastic (or random) process. When $T = N$, $\{ X_t: t \in T \}$ is said to be a discrete-time process and when $T = [0, \infty)$ it is called a continuous-time process, (Beichelt, 2016).

And stochastic processes are divided into two types.

A stochastic process with discrete parameter space (time) $\{X_n: n = 0, 1, 2, \ldots\}$.

A stochastic process in a continuous parameter space (time) $\{X_t: 0 \le t < \infty\}$, $t \in T$.

Markov process: It is the method by which the current changes of a given random variable are analyzed in order to predict future variables (Taj *et al*., 2007).

Where the conditional probability of X_{t_n} for a given set of values $\left\{X_{t_1}, X_{t_2}, X_{t_3}, ..., X_{t_{n-1}}\right\}$ depends only on $X_{t_{n-1}}$ any set of time periods $t_1, < t_2, < t_3, \ldots < t_n$, that is to say, the Markov process satisfies the following relationship:

$$P_r\left(X_{t_n} \le x_n \middle| X_{t_1} = x_1, X_{t_2} = x_2, ..., X_{t_{n-1}} = x_{n-1}\right) = P_r\left(X_{t_n} \le x_n \middle| X_{t_{n-1}} = x_{n-1}\right) \quad (1)$$

The probability of transition to a certain state in the future depends only on its current state, not on its old states.

This study will use a discrete-state Markov chain.

Discrete-state Markov Chain:

Definition (2): A random process $\{X_n : n \in T\}$ is called a discrete-state Markov chain if the following conditions are met (Kad, 1997).

The state space of this process is discrete.

The parameter space for this operation is discrete.

The process has the Markov property:

$$P_r\left(X_{n+1} = j \middle| X_n = i, X_{n-1} = i_{n-1}, ..., X_1 = i_1\right) = P_r\left(X_{n+1} = j \middle| X_n = i\right) \quad (2)$$

Definition (3): The n-step transition probability for discrete-state Markov chain is:

$$p_{ij}^{(n)} = P_r\left(X_{k+n} = j \middle| X_k = i\right) \quad (3)$$

Scott *et al*. **(**2012). and $p_{ij}^{(n)}$ to represent the probability of the system (random process) transition from state i to state j after a steps and it is called the probability of Transition in the n'th step.

Transition Matrix:

Definition (4): For a Markov chain $\{X_n: n \in T\}$ called a random matrix P of size $(n \times n)$, its elements are represented by the transition probabilities: $P = [p_{ij}]$ for each value $i, j \in T$ and written as:

$$P = \begin{array}{c} \\ i=1 \\ i=2 \\ \vdots \\ i=n \end{array} \begin{array}{c} \begin{array}{cccc} j=1 & j=2 & .... & j=n \end{array} \\ \begin{pmatrix} p_{11} & p_{12} & .... & p_{1n} \\ p_{21} & p_{21} & .... & p_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n2} & .... & p_{nn} \end{pmatrix} \end{array}$$

It is called a Markov matrix and the element whose arrangement (i, j) represents the probability p_{ij}, which is the probability of the system (random process) transiting from state i to state j in one step and the transition matrix satisfies the following conditions:

All elements of the matrix P are non-negative, that is $p_{ij} \ge 0 \ \forall i, j \in T$.

The sum of the elements of any of its rows is equal to one, that is. $\sum_{j \in T} p_{ij} = 1 \ \forall \ i \in T$

Classification of States

Recurrent State

Definition (5): A state i of the Markov chain $\{X_n\}$ is said to be a recurrent state if:

$$\sum_{n=1}^{\infty} P_r\left(X_n = i, X_{n-1} \ne i, X_{n-2} \ne i, ... ,, X_1 \ne i \middle| X_0 = i\right) = 1$$

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A state ***i*** is said to be a recurrent state if after starting at this state the chain is certain to return to it again.

Transient State

Definition (6): A state i of the Markov chain $\{X_n\}$ is said to be a transient state if and only if any starting from that state is not certain to return to it again.

That is, if $\{X_n\}$ a Markov chain is in state space S, the state $i \in S$ is called transient if:

$$\sum_{n=1}^{\infty} P_r\left(X_n = i,\, X_{n-1} \neq i,\, X_{n-2} \neq i, \dots,,\, X_1 \neq i \middle| X_0 = i\right) < 1$$

A transient state is a state which is not recurrent.

Absorbing State

Definition (7): A state i of the Markov chain $\{X_n\}$ is said to be an absorbing state, if the chain enters this state, it will remain there forever. That is to say probability of leaving is zero, meaning that:

$$p_{ij} = \begin{cases} 1 & ; i = j \\ 0 & ; i \neq j \end{cases}$$

An absorbing Markov chain is a Markov chain, which has an absorbing state and in which every state can reach an absorbing state and an absorbing state is a state that, once entered, cannot be left.

And the data used in the study are the data of students who study and move from one stage to another, as well as the graduation status and the cases of transition from one academic level to a higher level represent transient cases and the graduation case is the absorbent state.

For a Markov chain with ***k*** absorbing states and l transient states the transition matrix can be written in the form:

$$P = \begin{pmatrix} \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,k} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,k} \\ \vdots & \vdots & \cdots & \vdots \\ p_{k,1} & p_{k,2} & \cdots & p_{k,k} \end{pmatrix} & \begin{pmatrix} p_{1,k+1} & p_{1,k+2} & \cdots & p_{1,k+\ell} \\ p_{2,k+1} & p_{2,k+2} & \cdots & p_{2,k+\ell} \\ \vdots & \vdots & \cdots & \vdots \\ p_{k,k+1} & p_{k,k+2} & \cdots & p_{k,k+\ell} \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \end{pmatrix}$$

In short, the previous transition matrix can be written as:

$$P = \begin{pmatrix} Q & R \\ O & I \end{pmatrix} \qquad ;\ I_{\ell\times\ell}\ , O_{\ell\times k}\ , R_{k\times\ell}\ , Q_{k\times k}$$

$R_{k\times t}$ is a matrix that includes the probabilities of moving from transient states to absorption states and it is possible to only one stage (graduation states), that is:

$$R_{k\ell} = \begin{pmatrix} p_{1,k+1} & p_{1,k+2} & \cdots & p_{1,k+\ell} \\ p_{2,k+1} & p_{2,k+2} & \cdots & p_{2,k+\ell} \\ \vdots & \vdots & \cdots & \vdots \\ p_{k,k+1} & p_{k,k+2} & \cdots & p_{k,k+\ell} \end{pmatrix}$$

$Q_{k\times k}$ is a matrix that includes the probabilities of moving from (non-absorbing) to any (non-absorbing), or it can be said from transient states to transient states as well, that is:

$$Q_{kk} = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,k} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,k} \\ \vdots & \vdots & \cdots & \vdots \\ p_{k,1} & p_{k,2} & \cdots & p_{k,k} \end{pmatrix}$$

By finding the powers of the Transition Matrix, we find that:

$$P^n = \begin{pmatrix} Q^n & R\sum_{i=0}^{n-1} Q^i \\ O & I \end{pmatrix}$$

and that the limit of the matrix powers when $n \to \infty$ is:

$$\lim_{n\to\infty} P^n = \begin{pmatrix} O & R(I-Q)^{-1} \\ O & I \end{pmatrix}$$

This means that:

(1) $Q^n \to 0 \quad ; n \to \infty$

This means that in the long run, the probability of transition from the transient state to the transient state is also zero. That it is to say it will be a certain probability of the process transition from the transient state to the absorbing state:

(2) $\sum_{i=0}^{\infty} Q^i = (I-Q)^{-1}$

This represents the basic matrix of the Markov absorbing chain.

Each element of column i of the matrix $R(I - Q)^{-1}$ represents the probability of completion for each case of absorption when the process started in the transient state i.

And $(I - Q)^{-1}$ to give the expected number of times the process is in a transient state as it started in a transient state j.

That is to say, the sum of the column i from $(I - Q)^{-1}$ gives the expected number of times assuming that the process will be in a transient state since it started with a transient state j.

Application

In this part, we study the movement of student numbers according to the Markov chain model represented by the number of students moving from any stage to another during the study program and during the study period (2013-2019) and converting it to a Markov chain, where this case represents the number of students in each stage of the transition, then Constructing the transition matrix for the purpose of creating the transition matrix, which represents the transition matrix from any stage to another in one unit of time (an academic year).

The parameter and state spaces of a Markov chain. In this case: The parameter space of this chain is the set T = {2013/2014, 2014/2015, 2015/2016, 2016/2017, 2017/2018, 2018/2019} and the state space of this chain is the set S = {1st year, 2nd year, 3rd year, 4th year, 5th year, graduates}.

The number of academically withdrawn and dismissed students each academic year during the study period.

The number of students transferred to and from the Faculty of Dentistry each academic year.

The total number of students in each academic year consists of all academic movements in the same academic year.

The Transition Matrix

The transitional matrix (Markov movement of students) is represented by the movement of students' groups during the years of study at the Faculty of Dentistry, which will be symbolized with the symbol P. It is a matrix of transition probabilities and students' movement during the time of obtaining the required degree. The matrix of transition of college students will be organized according to the number of years given to obtain the degree and gender (male - female).

The Matrix of student movement in the College of Dentistry: It is a matrix of size (8 x 8) because there are five cases of student transfer in the years of study in the college representing the movement of students and their movement in the years and stages of the study program. In addition, there are three cases (the student's transfer to other colleges k_I), Or a case (withdrawal from study or academic dismissal k_{II}) and the third case is a case (the student graduated from the college after passing the academic program in the college k_{III}) and thus the Markov transition matrix for the movement of students at the College of Dentistry is formed as:

$$P = \left[p_{ij} \right]; i = j = 1,2,3,4,5, I, II, III$$

The previous matrix represents the movement of students between the different academic levels (according to the college's timetable) from failure cases, academic dismissals, or transfers to and from the college.

The study will employ the design of the Markov matrix for the movement of students in the college as follows:

$$P = \begin{pmatrix} Q & R \\ O & I \end{pmatrix}; I_{2\times2} \ , O_{2\times6} \ , R_{6\times2} \ , Q_{6\times6}$$

To form the Markov matrix of students' movement in the Faculty of Dentistry, we have the following tables:

It is clear from the table that the annual graduation rate is as follows:

Where the average was calculated by the relationship:

$$Annual\ Graduation\ Rate = \frac{the\ number\ of\ students\ graduating}{number\ of\ fifth\ year\ students}$$

Annual graduation rate for both genders (males + females) = 0.915

Annual graduation rate (male) = 0.969 and (female) = 0.890.

From the data of Table (1) and (2), we notice the following:

$$\begin{aligned} &The estimated\ probability that\ the student\ will \\ &remain\ (fail)\ at\ stage\ i = \frac{Noof\ students\ who\ have\ failed\ in\ stage\ i}{Noof\ regular\ students\ in\ stage\ i} \end{aligned}$$

The following table shows the odds of remaining (failure) at any stage:

From the data of Tables (1) and (4), we observe the following:

$$\begin{aligned} &The estimated\ probability\ of\ a\ student\ transferring \\ &to\ college\ at\ stage\ i \frac{Number\ of\ transferred\ students\ in\ stage\ i}{Number\ of\ regular\ students\ in\ stage\ i} \end{aligned}$$

The following table shows the estimated probability of conversion at any stage:

From the data of Tables (1) and (6), we notice the following:

$$\text{The estimated probability of a student being dismissed or with drawing from the college at stage } i = \frac{\text{Number of students dismissed or withdrawn from stage } i}{\text{Number of regular students in stage } i}$$

The following table shows the estimated probability of dismissal and withdrawal in any stage:

From the results of the analysis in Table (3), (5), (7), (8) the probability of the student succeeding in any stage *i* and transition to stage (*i*+1) can be obtained as follows:

$$P_{sum} = \begin{array}{c} \\ k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_I \\ k_{II} \\ k_{III} \end{array} \begin{array}{c} \begin{array}{cccccccc} k_1 & k_2 & k_3 & k_4 & k_5 & k_I & k_{II} & k_{III} \end{array} \\ \begin{pmatrix} 0.170 & 0.743 & 0 & 0 & 0 & 0 & 0.087 & 0 \\ 0 & 0.150 & 0.808 & 0 & 0 & 0 & 0.042 & 0 \\ 0 & 0 & 0.078 & 0.890 & 0 & 0 & 0.032 & 0 \\ 0 & 0 & 0 & 0.042 & 0.948 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0.012 & 0 & 0 & 0.988 \\ 0.003 & 0.024 & 0.0 & 0.003 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \quad ....(M-1)$$

And the transition matrix of the gender (female) as follows:

$$P_{fmale} = \begin{array}{c} \\ k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_I \\ k_{II} \\ k_{III} \end{array} \begin{array}{c} \begin{array}{cccccccc} k_1 & k_2 & k_3 & k_4 & k_5 & k_I & k_{II} & k_{III} \end{array} \\ \begin{pmatrix} 0.106 & 0.836 & 0 & 0 & 0 & 0 & 0.058 & 0 \\ 0 & 0.128 & 0.853 & 0 & 0 & 0 & 0.019 & 0 \\ 0 & 0 & 0.066 & 0.926 & 0 & 0 & 0.009 & 0 \\ 0 & 0 & 0 & 0.025 & 0.968 & 0 & 0.007 & 0 \\ 0 & 0 & 0 & 0 & 0.005 & 0 & 0 & 0.995 \\ 0.005 & 0.017 & 0.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \quad ....(M-2)$$

And the transition matrix of the type (male) as follows:

$$P_{male} = \begin{array}{c} \\ k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_I \\ k_{II} \\ k_{III} \end{array} \begin{array}{c} \begin{array}{cccccccc} k_1 & k_2 & k_3 & k_4 & k_5 & k_I & k_{II} & k_{III} \end{array} \\ \begin{pmatrix} 0.262 & 0.609 & 0 & 0 & 0 & 0 & 0.129 & 0 \\ 0 & 0.189 & 0.731 & 0 & 0 & 0 & 0.080 & 0 \\ 0 & 0 & 0.100 & 0.826 & 0 & 0 & 0.074 & 0 \\ 0 & 0 & 0 & 0.076 & 0.906 & 0 & 0.018 & 0 \\ 0 & 0 & 0 & 0 & 0.026 & 0 & 0 & 0.974 \\ 0 & 0.035 & 0 & 0.009 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \quad ....(M-3)$$

Now we can get a matrix to estimate the stay time in any of the non-absorbing cases, i.e., the student's stay in the faculty of dentistry (total - males - females), the matrix is: (I - Q).

where: $P = \begin{pmatrix} Q & R \\ O & I \end{pmatrix}$ and from matrices $(M-1)$, $(M-2)$, $(M-3)$ we can get:

$$Q_{sum},\ Q_{male},\ Q_{fmale}\ ,\ R_{sum}, R_{male}, R_{fmale}:$$

$$Q_{sum} = \begin{pmatrix} 0.170 & 0.743 & 0 & 0 & 0 & 0 \\ 0 & 0.150 & 0.808 & 0 & 0 & 0 \\ 0 & 0 & 0.078 & 0.890 & 0 & 0 \\ 0 & 0 & 0 & 0.042 & 0.948 & 0 \\ 0 & 0 & 0 & 0 & 0.012 & 0 \\ 0.003 & 0.024 & 0.0 & 0.003 & 0 & 0 \end{pmatrix}$$

$$Q_{fmale} = \begin{pmatrix} 0.106 & 0.836 & 0 & 0 & 0 & 0 \\ 0 & 0.128 & 0.853 & 0 & 0 & 0 \\ 0 & 0 & 0.066 & 0.926 & 0 & 0 \\ 0 & 0 & 0 & 0.025 & 0.968 & 0 \\ 0 & 0 & 0 & 0 & 0.005 & 0 \\ 0.005 & 0.017 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Q_{male} = \begin{pmatrix} 0.262 & 0.609 & 0 & 0 & 0 & 0 \\ 0 & 0.189 & 0.731 & 0 & 0 & 0 \\ 0 & 0 & 0.100 & 0.826 & 0 & 0 \\ 0 & 0 & 0 & 0.076 & 0.906 & 0 \\ 0 & 0 & 0 & 0 & 0.026 & 0 \\ 0 & 0.035 & 0 & 0.009 & 0 & 0 \end{pmatrix}$$

And:

$$R_{sum} = \begin{pmatrix} 0.087 & 0.042 & 0.032 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.988 & 0 \end{pmatrix}'$$

$$R_{fmale} = \begin{pmatrix} 0.058 & 0.019 & 0.009 & 0.007 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.995 & 0 \end{pmatrix}'$$

$$R_{male} = \begin{pmatrix} 0.129 & 0.080 & 0.074 & 0.018 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.974 & 0 \end{pmatrix}'$$

From the results of the matrices, $(M-1)$, $(M-2)$, $(M-3)$ we find:

$$(I-Q)^{-1}_{sum} = \begin{pmatrix} 1.2048 & 1.0574 & 0.9267 & 0.8609 & 0.8260 & 0 \\ 0.0000 & 1.1765 & 1.0310 & 0.9578 & 0.9190 & 0 \\ 0.0000 & 0.0000 & 1.0846 & 1.0076 & 0.9668 & 0 \\ 0.0000 & 0.0000 & 0.0000 & 1.0438 & 1.0016 & 0 \\ 0 & 0 & 0 & 0 & 1.0121 & 0 \\ 0.0036 & 0.0314 & 0.0275 & 0.0287 & 0.0275 & 1 \end{pmatrix} \quad ....(M-4)$$

$$(I-Q)^{-1}_{fmale} = \begin{pmatrix} 1.1186 & 1.0724 & 0.9794 & 0.9302 & 0.9049 & 0 \\ 0.0000 & 1.1468 & 1.0473 & 0.9947 & 0.9677 & 0 \\ 0.0000 & 0.0000 & 1.0707 & 1.0169 & 0.9893 & 0 \\ 0.0000 & 0.0000 & 0.0000 & 1.0256 & 0.9978 & 0 \\ 0 & 0 & 0 & 0 & 1.0050 & 0 \\ 0.0056 & 0.0249 & 0.0227 & 0.0216 & 0.0210 & 1 \end{pmatrix} \quad ....(M-5)$$

$$(I-Q)^{-1}_{male} = \begin{pmatrix} 1.3550 & 1.0175 & 0.8264 & 0.7388 & 0.6872 & 0 \\ 0 & 1.2330 & 1.0015 & 0.8953 & 0.8328 & 0 \\ 0 & 0 & 1.1111 & 0.9933 & 0.9239 & 0 \\ 0 & 0 & 0 & 1.0823 & 1.0067 & 0 \\ 0 & 0 & 0 & 0 & 1.0267 & 0 \\ 0 & 0.0432 & 0.0351 & 0.0411 & 0.0382 & 1 \end{pmatrix} \quad ....(M-6)$$

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The average absorption times can be obtained from the non-absorbing states; $N_{type} = (I - Q)^{-1}.[111111]'$:

$$N_{sum} = (4.8758 \quad 4.0843 \quad 3.0590 \quad 2.0454 \quad 1.0121 \quad 1.1188)' \quad ....(M-7)$$

$$N_{fmale} = (5.0054 \quad 4.1565 \quad 3.0768 \quad 2.0235 \quad 1.0050 \quad 1.0957)' \quad ....(M-8)$$

$$N_{male} = (4.6250 \quad 3.9626 \quad 3.0283 \quad 2.0889 \quad 1.0267 \quad 1.1575)' \quad ....(M-9)$$

Table 1: Number of students registered in the faculty of dentistry (new + regular) for the period 2013-2019

Academic year	newbies		1st year		2nd Year		3rd year		4th year		5th year		Graduates	
	M	F	M	F	M	F	M	F	M	F	M	F	M	F
2014/2013	54	72	54	65	51	86	50	89	44	85	27	71	27	71
2015/2014	60	98	52	89	37	74	32	65	35	61	25	63	25	63
2016/2015	62	83	49	83	54	87	34	64	24	61	35	61	34	61
2017/2016	61	76	49	69	48	84	50	87	30	65	28	54	27	54
2018/2017	54	72	54	65	51	86	50	89	44	85	27	71	27	71
2019/2018	67	69	44	61	48	53	54	76	48	86	49	89	45	44
	358	470	302	432	289	470	270	470	225	443	191	409	185	364

Source: College of dentistry - Ibb university

Table 2: The number of students who failed in the Faculty of Dentistry in the period 2013-2019

Academic year	1st year		2nd Year		3rd year		4th year		5th year		graduates	
	M	F	M	F	M	F	M	F	M	F	M	F
2014/2013	0	0	5	2	9	11	2	1	1	0	0	0
2015/2014	1	1	12	9	17	4	2	2	0	2	1	1
2016/2015	1	1	13	10	4	9	8	11	5	6	1	1
2017/2016	0	0	17	8	8	12	11	8	9	2	0	0
2018/2017	0	0	5	2	9	11	2	1	1	0	0	0
2019/2018	3	0	27	15	7	13	2	8	1	1	3	0
	5	2	79	46	54	60	27	31	17	11	5	2

Source: College of dentistry - Ibb university

Table 3: The estimated probability of staying (failing) in the faculty of dentistry in the period 2013-2019

	The estimated probability that the student will remain (fail) in the stage *i* Sum	Female	Male	The stage *i*	Example: For sum of types
$q_{k+1,k+1}$	0.170	0.106	0.262	1	$q_{k+1,k+1} = \frac{79+46}{302+432} = 0.170$
$q_{k+2,k+1}$	0.150	0.128	0.189	2	
$q_{k+3,k+1}$	0.078	0.066	0.100	3	
$q_{k+4,k+1}$	0.042	0.025	0.076	4	
$q_{k+5,k+1}$	0.012	0.005	0.026	5	

Source: Researcher

Table 4: Number of students transferred to the faculty of dentistry in the period 2013-2019

Academic year	1st year		2nd year		3rd year		4th year		5th year		graduates	
	M	F	M	F	M	F	M	F	M	F	M	F
2013/2014	0	0	0	0	4	3	0	0	0	0	0	0
2014/2015	0	0	0	0	3	1	0	0	0	0	0	0
2015/2016	0	0	0	2	0	1	0	0	0	0	0	0
2016/2017	0	0	0	0	0	0	0	0	0	0	0	0
2017/2018	0	0	0	0	3	3	0	0	0	0	0	0
2018/2019	0	0	0	0	0	0	0	0	2	0	0	0
Σ	0	0	0	2	10	8	0	0	2	0	0	0

Source: College of dentistry - Ibb University

Table 5: The estimated probability of transferring to the faculty of dentistry in the period 2013-2019

	the estimated probability Transferring the student to the college in the stage *i* Sum	Female	Male	stage *i*	Example: for sum of types
$q_{k+1,1}$	0.003	0.005	0	1	$q_{k+1,2} = \frac{10+8}{289+470} = 0.024$
$q_{k+1,2}$	0.024	0.017	0.035	2	
$q_{k+1,3}$	0	0	0	3	
$q_{k+1,4}$	0.003	0	0.009	4	
$q_{k+1,5}$	0	0	0	5	

Source: Researcher

Table 6: Number of students dismissed or withdrawn from the faculty of dentistry in the period 2013-2019

	1st year		2nd year		3rd year		4th year		5th year		Graduates	
Academic year	M	F	M	F	M	F	M	F	M	F	M	F
2013/2014	0	0	10	5	2	1	6	0	1	0	0	0
2014/2015	0	0	3	4	4	3	0	0	0	0	0	0
2015/2016	0	0	8	3	11	2	1	1	0	2	0	0
2016/2017	0	0	3	3	0	0	1	3	2	1	0	0
2017/2018	0	0	10	5	2	1	6	0	1	0	0	0
2018/2019	0	0	5	5	4	2	6	0	0	0	0	0
Σ	0	0	39	25	23	9	20	4	4	3	0	0

Source: College of Dentistry - Ibb University

Table 7: The estimated probability of dismissal or withdrawal from the faculty of dentistry in the period 2013-2019

	The estimated probability of a student being dismissed or withdrawing from the college at stage i Sum	Female	Male	stage *i*	Example: For male of types
$p_{1,k+1}$	0.087	0.058	0.129	1	$p_{1,k+3} = \frac{20}{270} = 0.074$
p_{1k+2}	0.042	0.019	0.080	2	
$p_{1,k+3}$	0.032	0.009	0.074	3	
$p_{1,k+4}$	0.010	0.007	0.018	4	
$p_{1,k+5}$	0	0	0	5	

Source: Researcher

Table 8: The estimated probability of success in each stage of the College of Dentistry for the period 2013-2019

	The estimated probability of the student succeeding in stage *i* and its transition to *i*+1 Sum	stage *i* and its transition to *i*+1 Female	Male	For example:
Q_{12}	0.743	0.836	0.609	1:2 $Q_{1,2} = 1-\left(q_{k+1,k+1}+p_{1,k+1}\right) = 1-(0.170+0.087) = 0.743$
Q_{23}	0.808	0.853	0.731	2:3
Q_{34}	0.890	0.926	0.826	3:4
Q_{45}	0.948	0.968	0.906	4:5
Q_{5g}	0.988	0.995	0.974	5 to graduation

From the results of data analysis in the previous tables, we deduce the transition matrix for the total of students, as follows:

It can also predict the number of students expected to obtain a bachelor's degree in dentistry in the five years following the study period, by constructing the following matrices:

$$E_{sum} = \left(\text{I-Q}\right)_{sum}^{-1}.R_{sum} \quad , \quad E_{fmale} = \left(\text{I-Q}\right)_{fmale}^{-1}.R_{fmale} \quad , \quad E_{male} = \left(\text{I-Q}\right)_{male}^{-1}.R_{male}$$

And via these matrices:

$(M-1)$, $(M-2)$, $(M-3)$, $(M-4)$, $(M-5)$, $(M-6)$ we find:

$$E_{sum} = \begin{pmatrix} 0.1875 & 0.0920 & 0.0448 & 0.0104 & 0 & 0.0028 \\ 0.8161 & 0.9080 & 0.9552 & 0.9896 & 1 & 0.0272 \end{pmatrix}' \quad ...(M-10)$$

$$E_{fmale} = \begin{pmatrix} 0.1006 & 0.0382 & 0.0168 & 0.0072 & 0 & 0.0012 \\ 0.9004 & 0.9629 & 0.9843 & 0.9928 & 1 & 0.0209 \end{pmatrix}' \quad ...(M-11)$$

$$E_{male} = \begin{pmatrix} 0.3307 & 0.1889 & 0.1001 & 0.0195 & 0 & 0.0068 \\ 0.6693 & 0.8111 & 0.8999 & 0.9805 & 1 & 0.0372 \end{pmatrix}' \quad ...(M-12)$$

Also, from the data in Table (1), the average enrollment in the five different academic years can be obtained as follows:

Academic year	1st year ---------------- M	F	2nd Year -------------- M	F	3rd year ----------- M	F	4th year ------------ M	F	5th year ---------- M	F
	60	86	58	94	54	94	45	89	38	82
	146	152	148	134	120					

From the previous table we get the following vectors:

$$M_{sum} = \begin{pmatrix}146 & 152 & 148 & 134 & 120\end{pmatrix}$$
$$M_{male} = \begin{pmatrix}60 & 58 & 54 & 45 & 38\end{pmatrix}$$
$$M_{fmale} = \begin{pmatrix}86 & 94 & 94 & 89 & 82\end{pmatrix}$$

Now, the number of students expected to obtain a bachelor's degree in dentistry in the five years following the study period can be predicted by multiplying the vectors M_{sam}, M_{fmde}, M_{mde} by the matrices M_{sum}, M_{fmde}, M_{mde} as follows:

$$F_{sum} = \begin{pmatrix}146 & 152 & 148 & 134 & 120\end{pmatrix} \begin{pmatrix} .1875 & .0920 & .0448 & .0104 & 0 \\ .8161 & .9080 & .9552 & .9896 & 1 \end{pmatrix}'$$
$$\therefore F_{sum} = \begin{pmatrix}49.4 & 651.1\end{pmatrix} \quad ...(M-13)$$

$$F_{fmale} = \begin{pmatrix}86 & 94 & 94 & 89 & 82\end{pmatrix} \begin{pmatrix} .1006 & .0382 & .0168 & .0072 & 0 \\ .9004 & .9629 & .9843 & .9928 & 1 \end{pmatrix}'$$
$$\therefore F_{fmale} = \begin{pmatrix}14.5 & 430.8\end{pmatrix} \quad ...(M-14)$$

$$F_{male} = \begin{pmatrix}60 & 58 & 54 & 45 & 38\end{pmatrix} \begin{pmatrix} .3307 & .1889 & .1001 & .0195 & 0 \\ .6693 & .8111 & .8999 & .9805 & 1 \end{pmatrix}'$$
$$\therefore F_{male} = \begin{pmatrix}37.1 & 217.9\end{pmatrix} \quad ...(M-15)$$

Discussion and Conclusion

Based on the previous results of the operations on the transfer matrices for students' movement in the College of Dentistry, the study presents the following results.

1. From the results of the matrices: (M - 7), (M - 8), (M - 9) we find that the average time spent by a student in the College of Dentistry to obtain a bachelor's degree is a ordinary average time for obtaining a bachelor's degree (graduation) for the total students although the average times for the male type are less than the average times for the female type and in an inverse form of the first years to the last years, we find that the average expected time for obtaining a bachelor's degree (graduation) for males in the first years is less than the average expected time for females in the first years and vice versa for the students in the last years as follows:

$$N_{sum} = \begin{pmatrix}4.88 & 4.08 & 3.06 & 2.05 & 1.01\end{pmatrix}'$$
$$N_{fmale} = \begin{pmatrix}5.01 & 4.16 & 3.08 & 2.02 & 1.01\end{pmatrix}'$$
$$N_{male} = \begin{pmatrix}4.63 & 3.96 & 3.03 & 2.09 & 1.03\end{pmatrix}'$$

2. From the results of the matrices: (M - 13), (M - 14), (M - 15) we find that the expected number of students who will obtain a bachelor's degree in dentistry in the five years following the study period as well as the expected number of students who will be dismissed from the study for the same period are as follows.

The number of students expected to graduate	Sum Male	Female
The number of students expected to be disenrolled	651 218	431
	49 37	15

From the previous table, it is clear that the number of the female graduates is double the number of the male graduates. In contrast, the number of dismissed females is one third of the number of dismissed males and this is also a natural result of the female graduates' discipline and their commitment to rules and regulations.

This study can be generalized to include all dental faculties in different universities in the Republic of Yemen, compare their results and test the significance of differences, if any.

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Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

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