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Optimal Ordering Policy with Stock-Dependent Demand Rate Under Retailer's Two Stages Trade Credit Financing Using Discounted Cash Flow (DCF) Approach

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Abstract: Many researchers have assumed one stage trade credit financing. In this study, we considered two levels of trade credit policy using Discounted Cash Flow (DCF) approach. Demand rate is considered to be stock-dependent for the first level (credit demand) and constant for second level (cash demand). Mathematical models are derived under two different circumstances i.e., case I: The permissible delay period is less than or equal to the cycle time and case II: The permissible delay period is greater than or equal to the cycle time for settling the account. An algorithm is provided to determine the optimal order quantity and annual profit. In addition, numerical examples are presented to demonstrate the solution process. Finally, sensitivity analysis of the optimal solution is discussed with respect to different parameters.

Keywords: Discounted Cash Flow, Inventory, Two Level Credit Policy, Credit Linked Demand, Credit Period

Introduction

In classical Economic Order Quantity (EOQ) model, it is assumed that the supplier is paid for the items instantly after they are received. In practice, the supplier permits a certain fixed credit period to settle the account for invigorated retailer's demand. The permissible delay in payment is helpful to attract new customer and increase sales. Inventory models with credit period were first developed by Goyal (1985) to push aside the difference between the selling price and purchase cost. Dave (1985) modified and extended Goyal (1985) model adding the fact that the selling price is necessarily higher than its purchase cost. Haley and Higgins (1973) established the first model to consider the economic order quantity under conditions of permissible delay in payment with deterministic demand. Shah (1993) considered a stochastic inventory model when delays in payments are permissible. Aggarwal and Jaggi (1995) modified Goyal (1985) model for deteriorating items. Jamal *et al.* (1997) further extended model (1995) allow for shortages. Chang *et al*. (2003) developed an EOQ model under supplier credits linked to ordering quantity for deteriorating items. Chung and Huang (2003) presented an Economic Production Quantity (EPQ) model for a retailer where the supplier offers a permissible delay in payments. Teng *at al*. (2012) presented an EOQ model under trade credit financing with increasing demand. Khanra *et al*. (2011) developed an EOQ model for time dependent demand when delay in payment is permissible. Many researchers like Chu *et al*. (1998; Chung *et al*., 2001; Davis and Gaither, 1985; Mandal and Phaujdar, 1989a; Chang *et al*., 2001; Chung and Liao, 2004; Saiedy and Moghadam, 2011) worked on inventory model by considering delay in payment. Ouyang *et al*. (2004) presented an inventory model with non instantaneous receipt under permissible delay in payments. Jaggi *et al*. (2007) developed the retailer's optimal ordering policy under two stage trade credits financing using Discounted Cash Flow (DCF) approach.

In real world, the consumption rate is sometimes affected by the stock level. It is usually observed that a large pile of items on large rack in a supermarket will show the customer to purchase more and then generate demand. The consumption rate may fluctuate with the on hand inventory. Yang *et al*. (2010) presented an inventory model for deteriorating item with stock-dependent demand and partial backlogging. Soni and Shah (2008) established inventory model for retailer when demand is partially constant and partially dependent on the stock and the supplier offers progressive credit periods to settle the account. Teng *et al*. (2011) modified and extended the model (2008) for different situations. Mandal and Phaujdar (1989b) developed a production stock-dependent demand. Two closely related research papers/ articles on stock-dependent rate published by Chang *et al*. (2010). Alfares (2007) established inventory models in which the demand rate depends on the inventory level and storage time-dependent holding cost. Pal *et al*. (1991) developed a deterministic inventory model assuming that the demand rate is stock-dependent for deteriorating items. Silver and Peterson (1985) observed that a sale at the retail level is proportional to the amount of inventory displayed. Gupla and Vrat (1986) established inventory model in which demand rate to be a function of initial stock level. Some of the related research in this area are by Wee (1995; Goh, 1994; Ray and Chaudhuri, 1997; Mandal and Maiti, 1999; Dye, 2002; Chung and Tsai, 2001; Yan and Cheng, 1998; Sarker *et al*., 1997) etc.

At present the effect of inflation plays an important role in any type of business. At present developing countries are facing large scale of inflation due to lock off, strike, natural calamities, political disturbances etc. Thus the effect of inflation cannot be disregarded in real word. Hou (2006) derived an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time value of money discounting over a finite planning horizon. Ouyang *et al*. (2002) studied the thump of trade credit in the inventory system. Hou and Lin (2009) developed an inventory model to determine an optimal ordering policy for deteriorating item with delayed payment permitted by the supplier under-inflation and time discounting. Other related research papers/articles were considered by Chang (2004; Chung and Liao, 2006; Jaggi and Aggarwal, 1994; Chapman *et al*., 1985; Daellenbach, 1986; Haley and Higgins, 1973). Jaggi *et al*. (2007) determined the retailer's optimal ordering policy under two stage trade credits financing using Discounted Cash Flow (DCF) approach.

Jaggi *et al*. (2007) developed an inventory model under two levels of trade credit policy by assuming the demand is a function of credit period offered by the retailer to the customer using Discounted Cash Flow (DCF) approach. In this study an attempt is made to formulate the mathematical model for stock-dependent credit demand and constant cash demand. The objective function to be maximized is appraised as the retailer's net profit of the inventory system. The effect of parameters on the objective function is discussed numerically. An algorithm is provided to validate the proposed model.

The rest of the paper organized as follows. In the next section, we provide the notations and assumptions for the proposed model. Mathematical formulation is established to manifest retailer's net profit in section 3. Section 4, provides the optimal solution for finding optimal cycle time. In section 5, algorithm is developed for finding optimal solution. Numerical examples are provided to illustrate the solution algorithm in section 6. In section 7, sensitivity analysis of the optimal solution with respect to different parameters of the system is carried out. Finally, we draw the conclusion and future research in section 8.

Notations and Assumptions

The following notations are used through the manuscript:

- $I(t)$: The inventory level time 't'
- Q: The order quantity
- S: The ordering cost per order at time zero
- c: The unit purchase cost of the item at time zero
- p: The unit selling price of the item at time zero
- i: Out-of-pocket inventory carrying charge per \$ per year
- r: Discount rate per year
- I_e: The interest that can be earned per \$ per year
- I_p: The interest charges payable per dollar per year ($I_p > I_e$)
- m: Credit period offered to retailer by the supplier for settling the accounts
- T_1: Credit period granted by the retailer to his/ her customers; $T_1 \le m$
- T: The inventory cycle time in years
- T^*: Optimal inventory cycle time for case I in years
- T^{**}: Optimal inventory cycle time for case II in years
- $Z_1(T$: Retailer's annual net profit per cycle for case I
- $Z_2(T)$: Retailer's annual net profit per cycle for case II
- $Z_1^*(T^*)$: Optimal retailer's annual net profit per cycle for case I
- $Z_2^*(T^{**})$: Optimal retailer's annual net profit per cycle for case II
- Q: Order quantity
- Q_1^*: Optimal order quantity for case I
- Q_2^*: Optimal order quantity for case II

Assumptions:

In addition, the following assumption is being through manuscript:

- Replenishment rate is instantaneous
- Shortages are not allowed
- The annual demand rate consists of (a) regular cash demand and (b) credit demand. Thus demand function at time t is given by:

$$R(t)=\begin{cases}\alpha+\beta I(t) & ,0\le t\le T_1,\ 0<\beta\le 1\\ \alpha & ,T_1\le t\le T\end{cases}$$

where, α is known and constant cash-demand rate during the cycle $[0,T]$ and β is the credit demand rate during the customer's credit demand rate during the customer's credit period T_1:

- The model is considered for one item only
- The Discounted Cash Flow (DCF) approach is applied to consider the various the various cost at various times
- The supplier provides a credit period m to resolve the account to the retailer and retailer passes on a maximum credit T_1 to its customers to resolve the account. We assume $T_1\le m$ and customer would resolve their account only on last day of the credit period T_1

Mathematical Formulation

The inventory is depleted due to demand only. Thus, the rate of change of inventory at time t in $[0, T]$ is given by:

$$\frac{dI_1(t)}{dt}=-\{\alpha+\beta I_1(t)\},\ 0\le t\le T_1 \qquad (1)$$

$$\frac{dI_2(t)}{dt}=-\alpha,\ T_1\le t\le T \qquad (2)$$

The rate of change of inventory can be easily seen in Fig. 1.

With the boundary condition I(0) = Q , I(T) = 0, the solution of (1) and (2) is given by:

$$I_1(t)=\left(Q+\frac{\alpha}{\beta}\right)e^{-\beta t}-\frac{\alpha}{\beta} \qquad (3)$$

$$I_2(t)=\alpha(T-t) \qquad (4)$$

and

$$Q_1=\left(Q+\frac{\alpha}{\beta}\right)\left(1-e^{-\beta T_1}\right) \qquad (5)$$

$$Q_1=\alpha(T-T_1) \qquad (6)$$

Using (5) and (6) in (3), we get:

$$I_1(t)=\alpha\left\{\frac{e^{\beta(T_1-t)}-1}{\beta}+(T-T_1)e^{\beta(T_1-t)}\right\},\ 0\le t\le T_1 \qquad (7)$$

and

$$Q=Q_1+Q_2=\left(Q+\frac{\alpha}{\beta}\right)\left(1-e^{-\beta T_1}\right)+\alpha(T-T_1)$$

or

$$Q=\alpha\left\{\frac{e^{\beta T_1}-1}{\beta}+(T-T_1)e^{\beta T_1}\right\} \qquad (8)$$

By using the discounted cash flow approach, the different components of the retailer's net profit is calculated as follows:

The present value of the sales revenue is:

$$\begin{aligned}&=\frac{p}{T}\left\{\int_0^T \alpha e^{-rt}dt+e^{-rT_1}\int_0^{T_1}\beta I_1(t)dt\right\}\\ &=\frac{\alpha p}{T}\left[\frac{1-e^{-rT}}{r}+e^{-rT_1}\left\{\frac{e^{\beta T_1}-1}{\beta}+(T-T_1)e^{\beta T_1}-T\right\}\right]\end{aligned} \qquad (9)$$

$$\text{The present cost of placing order}=\frac{s}{T} \qquad (10)$$

$$\text{The ordering cost}=\frac{cQ}{T}=\frac{c\alpha}{T}\left\{\frac{e^{\beta T_1}-1}{\beta}+(T-T_1)e^{\beta T_1}\right\} \qquad (11)$$

The present cost of out of pocket inventory carrying cost is:

$$\begin{aligned}&=\frac{ic}{T}\left(\int_0^{T_1} I_1(t)e^{-rT}dt+\int_{T_1}^{T} I_2(t)e^{-rT}dt\right)\\ &=\frac{ic\alpha}{T}\left[\begin{array}{l}\left(\frac{1}{\beta}+T-T_1\right)\left(\frac{e^{\beta T_1}-e^{-rT_1}}{\beta+r}\right)+\frac{e^{-rT_1}-1}{\beta r}+\frac{1}{r}\\ \left\{(T-T_1)e^{-rT_1}+\frac{e^{-rT}-e^{-rT_1}}{r}\right\}\end{array}\right]\end{aligned} \qquad (12)$$

The following two cases arise which is based on the value of T and m.

Case I: $m\le T$

In this case, the retailer deposits the assembled revenue from cash sales in the period $[0,m]$ and also from credit sales in time period $[T_1,m]$ in to an account that earns interest rate I_e. At credit period 'm' credit period, the account have to be resolved, it is assumed that account will be fixed by proceeds of sells produced up to credit period m and by taking a short term credit at an interest rate of I_p in between $(T-m)$ for financing the remaining stock. Therefore, the present interest earned is:

$$\frac{pI_e}{T}\left[\int_0^m \alpha te^{-rt}dt + \int_{T_1}^m\left\{\beta\int I_1(t)dt\right\}e^{-rT}dt\right]$$
$$= \frac{\alpha pI_e}{T}\left[\begin{array}{l}\frac{1}{r^2}\left\{1-(1+rm)e^{-rm}\right\}+ \\ \left\{\frac{e^{\beta T_1}-1}{\beta}+(T-T_1)e^{\beta T_1}-T\right\}\left(\frac{e^{-rT_1}-e^{-rm}}{r}\right)\end{array}\right] \quad (13)$$

The present interest payable is:

$$= \frac{cI_p}{T}\int_m^T I_2(t)e^{-rt}dt = \frac{\alpha cI_p}{T}\int_m^T (T-t)e^{-rt}dt$$
$$= \frac{\alpha cI_p}{rT}\left\{(T-m)e^{-rm} + \frac{e^{-rT}-e^{-rm}}{r}\right\} \quad (14)$$

The retailer's net profit $Z_1(T)$ can be expressed as $Z_1(T)$ = Sales Revenue + interest earned-purchase cost-ordering Cost-inventory carrying cost-interest payable:

$$= \frac{\alpha p}{T}\left[\frac{1-e^{-rT}}{r} + \left\{e^{-rT_1}+\frac{I_e}{r}\left(e^{-rT_1}-e^{-rm}\right)\right\}\left\{\frac{e^{\beta T_1}-1}{\beta}+(T-T_1)e^{\beta T_1}-T\right\}\right.$$
$$\left. + \frac{I_e}{r}\left\{\frac{1-(1+rm)e^{-rm}}{r}\right\}\right] - \frac{s}{T} - \frac{c\alpha}{T}\left(\frac{e^{\beta T_1}-1}{\beta}+Te^{\beta T_1}-T_1e^{\beta T_1}\right)$$
$$-\frac{ic\alpha}{T}\left\{\left(\frac{1}{\beta}+T-T_1\right)\left(\frac{e^{\beta T_1}-e^{-rT_1}}{\beta+r}\right)+\frac{e^{-rT_1}-1}{\beta r}+\frac{1}{r}\left(Te^{-rT_1}-T_1e^{-rT_1}+\frac{e^{-rT}-e^{-rT_1}}{r}\right)\right\}$$
$$-\frac{\alpha cI_p}{rT}\left\{(T-m)e^{-rm}+\frac{e^{-rT}-e^{-rm}}{r}\right\} \quad (15)$$

Case II: $m \geq T$

In this case the credit period m is longer than or equal to cycle time T, therefore the retailer gets interest on each sales during the period $[0,m]$ and also on credit sales in between $[T_1,m]$ and pay no interest for the raw material in stock. The interest earned is:

$$= \frac{pI_e}{T}\left[\int_0^T \alpha.t.e^{-rT}dt + \int_T^m I_2(0)e^{-rT}dt + \int_{T_1}^m\left\{\int_0^T \beta I_1(t)dt\right\}e^{-rt}dt\right] = \frac{\alpha pI_e}{rT}\left\{\begin{array}{l}\frac{1-e^{-rT}-rTe^{-rm}}{r}+\left(\frac{e^{\beta T_1}-1}{\beta}+Te^{\beta T_1}-T_1e^{\beta T_1}-T\right) \\ \left(e^{-rT_1}-e^{-rm}\right)\end{array}\right\} \quad (16)$$

Therefore, the retailer's annual net profit $Z_2(T)$ is given by $Z_2(T)$ = Sales revenue + interest earned-purchase cost-ordering cost-cost of out of pocket inventory carrying cost:

$$= \frac{\alpha p}{T}\left[\left(\frac{1-e^{-rT}}{r}\right)\left(1+\frac{I_e}{r}\right)+\left\{e^{-rT_1}+\frac{I_e}{r}\left(e^{-rT_1}-e^{-rm}\right)\right\}\left\{\frac{e^{\beta T_1}-1}{\beta}+(T-T_1)e^{\beta T_1}-T\right\}\right.$$
$$\left.-\frac{I_eTe^{-rm}}{r}\right]-\frac{s}{T}-\frac{c\alpha}{T}\left\{\frac{e^{\beta T_1}-1}{\beta}+(T-T_1)e^{\beta T_1}\right\}-\frac{ic\alpha}{T}$$
$$\left[\left(\frac{1}{\beta}+T-T_1\right)\left(\frac{e^{\beta T_1}-e^{-rT_1}}{\beta+r}\right)+\frac{e^{-rT_1}-1}{\beta r}+\frac{1}{r}\left\{(T-T_1)e^{-rT_1}+\frac{e^{-rT}-e^{-rT_1}}{r}\right\}\right] \quad (17)$$

The present retailer's annual profit, $Z(T)$ can be expressed as:

$$Z(T) = \begin{cases} Z_1(T), & \text{if m} \leq \text{T} \\ Z_2(T), & \text{if m} \geq \text{T} \end{cases} \quad (18)$$

At $T = m$,
$Z_1(T) = Z_2(T)$

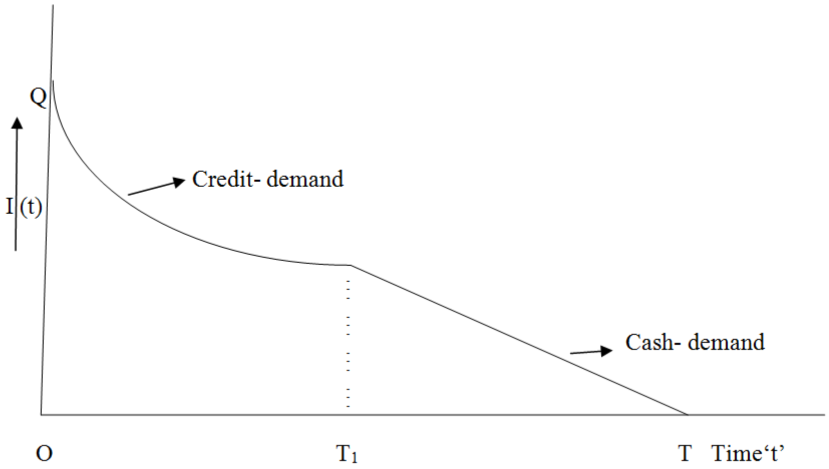

Fig. 1. I(t) Vs time

Determination of Optimal Solution

To determine optimal value of T, taking the first derivative of $Z_1(T)$ and $Z_2(T)$ with respect to T, we obtain:

$$\frac{dZ_1(T)}{dT} = -\frac{\alpha p}{T^2}\left[\frac{1-e^{-rT}}{r} - Te^{-rT} + \left\{e^{-rT_1} + \frac{I_e}{r}\left(e^{-rT_1} - e^{-rm}\right)\right\}\left(\frac{e^{\beta T_1}-1}{\beta} - T_1 e^{\beta T_1}\right)\right.$$
$$\left. + \frac{I_e}{r}\left\{\frac{1-(1+rm)e^{-rm}}{r}\right\}\right] + \frac{s}{T^2} + \frac{c\alpha}{T^2}\left(\frac{e^{\beta T_1}-1}{\beta} - T_1 e^{\beta T_1}\right) + \frac{ic\alpha}{T^2}\left\{\left(\frac{1}{\beta} - T_1\right)\left(\frac{e^{\beta T_1} - e^{-rT_1}}{\beta + r}\right)\right. \tag{19}$$
$$\left. + \frac{e^{-rT_1}-1}{\beta r} + \frac{1}{r}\left(Te^{-rT} - T_1 e^{-rT_1} + \frac{e^{-rT} - e^{-rT_1}}{r}\right)\right\} + \frac{\alpha c I_p}{rT^2}\left\{\left(T + \frac{1}{r}\right)e^{-rT} - \left(m + \frac{1}{r}\right)e^{-rm}\right\}$$

and

$$\frac{dZ_2(T)}{dT} = -\frac{\alpha p}{T^2}\left[\left(\frac{1-e^{-rT}}{r} - Te^{-rT}\right)\left(1 + \frac{I_e}{r}\right) + \left\{e^{-rT_1} + \frac{I_e}{r}\left(e^{-rT_1} - e^{-rm}\right)\right\}\left(\frac{e^{\beta T_1}-1}{\beta} - T_1 e^{\beta T_1}\right)\right] + \frac{s}{T^2} +$$
$$\frac{c\alpha}{T^2}\left(\frac{e^{\beta T_1}-1}{\beta} - T_1 e^{\beta T_1}\right) + \frac{ic\alpha}{T^2}\left[\left(\frac{1}{\beta} - T_1\right)\left(\frac{e^{\beta T_1} - e^{-rT_1}}{\beta + r}\right) + \frac{e^{-rT_1}-1}{\beta r} + \frac{1}{r}\left\{\left(T + \frac{1}{r}\right)e^{-rT} - \left(T_1 + \frac{1}{r}\right)e^{-rT_1}\right\}\right] \tag{20}$$

Our aim is to find maximum retailer's annual profit. The necessary and sufficient condition to maximize $Z_i(T)$; $i = 1$, 2, for a given value T are respectively $\frac{dZ_i(T)}{dT} = 0$ and $\frac{d^2 Z_i(T)}{dT^2} > 0$; i = 1,2. (Appendix).

Now $\frac{dZ_i(T)}{dT} = 0$; i = 1,2. , give the following equation in T:

$$\alpha p\left[\frac{1-e^{-rT}}{r} - Te^{-rT} + \left\{e^{-rT_1} + \frac{I_e}{r}\left(e^{-rT_1} - e^{-rm}\right)\right\}\left(\frac{e^{\beta T_1}-1}{\beta} - T_1 e^{\beta T_1}\right) + \frac{I_e}{r}\left\{\frac{1-(1+rm)e^{-rm}}{r}\right\}\right]$$
$$-s - c\alpha\left(\frac{e^{\beta T_1}-1}{\beta} - T_1 e^{\beta T_1}\right) - ic\alpha\left\{\left(\frac{1}{\beta} - T_1\right)\left(\frac{e^{\beta T_1} - e^{-rT_1}}{\beta + r}\right) + \frac{e^{-rT_1}-1}{\beta r} + \frac{1}{r}\left(Te^{-rT} - T_1 e^{-rT_1} + \frac{e^{-rT} - e^{-rT_1}}{r}\right)\right\} \tag{21}$$
$$-\frac{\alpha c I_p}{r}\left\{\left(T + \frac{1}{r}\right)e^{-rT} - \left(m + \frac{1}{r}\right)e^{-rm}\right\} = 0$$

and

$$\alpha p\left[\left(\frac{1-e^{-rT}}{r}-Te^{-rT}\right)\left(1+\frac{I_e}{r}\right)+\left\{e^{-rT_1}+\frac{I_e}{r}\left(e^{-rT_1}-e^{-rm}\right)\right\}\left(\frac{e^{\beta T_1}-1}{\beta}-T_1e^{\beta T_1}\right)\right]$$
$$-s-c\alpha\left(\frac{e^{\beta T_1}-1}{\beta}-T_1e^{\beta T_1}\right)-ic\alpha\left[\left(\frac{1}{\beta}-T_1\right)\left(\frac{e^{\beta T_1}-e^{-rT_1}}{\beta+r}\right)+\frac{e^{-rT_1}-1}{\beta r}+\frac{1}{r}\left\{\left(T+\frac{1}{r}\right)e^{-rT}-\left(T_1+\frac{1}{r}\right)e^{-rT_1}\right\}\right]=0 \quad (22)$$

To get the optimal cycle time $T = T^*$ for case I and $T = T^{**}$ for case II, we have to solve Equations (21) and (22), for which $\frac{d^2Z_i(T)}{dT^2}<0$. for i = 1, 2 (Appendix).

Since it is difficult to solve above Equations (21) and (22), for finding the exact value of T, therefore, we make use of the second order approximations for exponential terms, i.e., $e^{-rT}=1-rT+\frac{r^2T^2}{2}$, $e^{\beta T_1}=1-\beta T_1+\frac{\beta^2T_1^2}{2}$ and $e^{-rT_1}=1-rT_1+\frac{r^2T_1^2}{2}$ etc.

Hence Equations (21) and (22) reduces to:

$$\alpha p\left[\frac{r(1-rT)T^2}{2}-\frac{\beta T_1^2}{2}\left\{1-rT_1+\frac{r^2T_1^2}{2}+I_e(T_1-m)\left(\frac{rT_1}{2}+\frac{rm}{2}-1\right)\right\}(1+\beta T_1)+\frac{I_em^2}{2}\left(1-rm\right)\right]$$
$$-s+\frac{c\alpha\beta T_1^2}{2}\left(1+\beta T_1\right)+\frac{ic\alpha}{2}\left\{T_1^2\left(1+\beta T_1-rT_1\right)-(T-T_1)\right\}\left\{r(T^2+TT_1+T_1^2)-(T+T_1)\right\}$$
$$-\frac{\alpha cI_p}{2}\left\{m^2(1-rm)-T^2(1-rT)\right\}=0 \quad (23)$$

and

$$\alpha p\left[\frac{r(1-rT)T^2}{2}\left(1+\frac{I_e}{r}\right)-\frac{\beta T_1^2}{2}\left\{1-rT_1+\frac{r^2T_1^2}{2}+I_e(T_1-m)\left(\frac{rT_1}{2}+\frac{rm}{2}-1\right)\right\}(1+\beta T_1)\right]$$
$$-s+\frac{c\alpha\beta T_1^2}{2}\left(1+\beta T_1\right)+\frac{ic\alpha}{2}\left\{\beta T_1^3+T^2(1-rT)\right\}=0 \quad (24)$$

Again, we make use of the second order approximations for exponential terms, i.e., $e^{-rT}=1-rT+\frac{r^2T^2}{2}$, $e^{\beta T_1}=1-\beta T_1+\frac{\beta^2T_1^2}{2}$ and $e^{-rT_1}=1-rT_1+\frac{r^2T_1^2}{2}$ etc.

Hence Equations (8) (15) and (17) reduce to:

$$Q=\alpha\left\{T\left(1+\beta T_1+\frac{\beta^2T_1^2}{2}\right)-\frac{\beta T_1^2}{2}\left(1+\beta T_1\right)\right\} \quad (25)$$

$$Z_1(T)=\frac{\alpha p}{T}\left[T\left(1-\frac{rT}{2}\right)+\left\{1-rT_1+\frac{r^2T_1^2}{2}+I_e(T_1-m)\left(\frac{rT_1}{2}+\frac{rm}{2}-1\right)\right\}\frac{\beta T_1}{2}\left(2T-T_1+\beta TT_1-\beta T_1^2\right)+\frac{I_em^2}{2}(1-rm)\right]$$
$$-\frac{s}{T}-\frac{c\alpha}{T}\left\{\left(T-\frac{\beta T_1^2}{2}\right)(1+\beta T_1)+\frac{\beta^2T_1^2T}{2}\right\}-\frac{ic\alpha}{2T}\left\{T^2+\beta T_1^2(T-T_1)\right\}-\frac{\alpha cI_p(T-m)}{2T}\left\{T-m(1-rm)\right\} \quad (26)$$

and

$$Z_2(T)=\frac{\alpha p}{T}\left[T\left(1-\frac{rT}{2}\right)\left(1+\frac{I_e}{r}\right)+\left\{1-rT_1+\frac{r^2T_1^2}{2}+I_e(T_1-m)\left(\frac{rT_1}{2}+\frac{rm}{2}-1\right)\right\}\frac{\beta T_1}{2}\left(2T-T_1+\beta TT_1-\beta T_1^2\right)-\frac{I_eT}{r}\left(1-rm+\frac{r^2m^2}{2}\right)\right]$$
$$-\frac{s}{T}-\frac{c\alpha}{T}\left\{\left(T-\frac{\beta T_1^2}{2}\right)(1+\beta T_1)+\frac{\beta^2T_1^2T}{2}\right\}-\frac{ic\alpha}{2T}\left\{T^2+\beta T_1^2(T-T_1)\right\} \quad (27)$$

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Now, we summarize the above results and establish the following algorithm to find the optimal solution.

Algorithm

The following steps are to be followed to find optimal annual profit and order quantity:

Step 1: Determine T^* from (23), if $T \geq m$, evaluate $Z_1(T^*)$, from (26)
Step 2: Determine T^{**} from (24), if $T < m$, evaluate $Z_2(T^{**})$, from (27)
Step 3: If the condition $T^* \geq m$ and $T^{**} < m$ is satisfied, go to step 4, otherwise go to step 5
Step 4: Compare $Z_1(T^*)$ and $Z_2(T^*)$ and find the maximum profit
Step 5: If $T^* > m$ is satisfied but $T^{**} > m$, then $Z_1(T^*)$ the maximum profit, else if $T^* < m$, but $T^{**} < m$, then $Z_2(T^{**})$ is the maximum profit

Numerical Examples

Example 1: (Case I & II) Maximum Retailer's Annual Profit $Z_2^*(T^{**})$

The following data is considered for inventory system:

- α = 1000 units per year, β = 0.1, r = 13%, m = 5.0 year, T_1 =.5 year, c = \$20/ unit, i = 0.15, I_e = 9%, I_p = 14%, s = \$700/ unit, p = \$ 60 / unit. Solving Equation (23), we get $T = T^* = 7.67797$ years, the corresponding values of $Q = Q_1^* = 12087.5$ units and maximum retailer's annual profit $Z_1(T) = Z_1^*(T^*)$ = \$ 8058.34
- Again solving Equation (24), we have $T = T^{**}$ = 0.531856 year, the corresponding values of $Q = Q_2^*$ = 545.989 units and maximum retailer's annual $Z_2(T) = Z_2^*(T^{**})$ = \$ 53980.6
- Here $T^* > m$ and $T^{**} < m$ and $Z_1^*(T^*) < Z_2^*(T^{**})$. Hence the maximum average profit in this case is $Z_2^*(T^{**})$ = \$ 53980.6. Where optimal cycle time is $T = T^*$ = 0.531856 year
- The economic order quantity is $Q = Q_2^*$ = 545.989 units

Example 2: (case I) Maximum Retailer's Annual Profit $Z_1^*(T^*)$

The following data is considered for inventory system:

- α = 1000 units per year, β = 0.1, r = 13%, m = 0.0822 year, T_1 =0.0274 year, c = \$50/ unit, i = 0.15, I_e = 9%, I_p = 14%, s = \$500/ unit, p = \$ 60 / unit. Solving Equation (23), we get $T = T^*$ = 0. 225374 year, the corresponding values of $Q = Q_1^*$ = 225.955 units and maximum retailer's annual profit $Z_1(T) = Z_1^*(T^*)$ = \$ 5843.36
- Again solving Equation (24), we have $T = T^{**}$ = 0.431152, the corresponding values of $Q = Q_2^*$ = 432.297units and maximum retailer's annual $Z_2(T) = Z_2^*(T^{**})$ = \$ 4837.67
- Here $T^{**} > m$ which contradicts case II, only case I holds as $T^* > m$. Hence the maximum average profit in this case is $Z_1^*(T^*)$ = \$ 5843.36. Where optimal cycle time is $T = T^*$ = 0. 225374 year
- The economic order quantity is $Q = Q_1^*$ = 225.955 units

Example 3: (Case II) Maximum Retailer's Annual Profit $Z_2^*(T^{**})$

The following data is considered for inventory system:

- α = 1000 units per year, β = 0.1, r = 13%, m = 0.8 year, T_1 =0.4 year, c = \$50/ unit, i = 0.15, I_e = 9%, I_p = 14%, s = \$500/ unit, p = \$ 60/ unit. Solving Equation (23), we get $T = T^*$ = 0. 371669 year, the corresponding values of $Q = Q_1^*$ = 378.513 units and maximum retailer's annual profit $Z_1(T) = Z_1^*(T^*)$ = \$ 8439.39
- Again solving Equation (24), we have $T = T^{**}$ = 0.437502, the corresponding values of $Q = Q_2^*$ = 447.032 units and maximum retailer's annual profit $Z_2(T) = Z_2^*(T^{**})$ = \$ 8624.52
- Here T* < m which contradicts case I, only case II holds as $T^{**} < m$. Hence the maximum retailer's annual profit in this case is $Z_2^*(T^{**})$ = \$ 8624.52, where optimal cycle time is $T = T^{**}$ = 0.437502 year
- The economic order quantity is given by $Q = Q_2^*$ = 447.032 units

Sensitivity Analysis

By using the same data as in example 1, we study the effect of the changes in a single parameter keeping other parameters same on the optimal solution as shown in following Tables 1-8.

The following inferences can be made from the results obtained from Tables 1-8:

- When the cash demand 'α' increases, the order quantity (Q_1) and net profit $Z_1(T)$ will also increase. Similarly if the credit demand 'β' increases, the order quantity (Q_1) slightly increases and net profit $Z_1(T)$ increases. That is, change in 'α' will lead the positive change in Q_1 and $Z_1(T)$. The change in 'β' will lead slight change in Q_1 and change in $Z_1(T)$
- When the cash demand 'α' increases, order quantity (Q_1) and net profit $Z_1(T)$ will also increase. Similarly in purchase cost 'c' increases, the order quantity (Q_1) and net profit $Z_1(T)$ will also increase. That is, change in 'c' will lead the positive change in (Q_1) and $Z_1(T)$

- When the cash demand 'α' increases, order quantity (Q_1) and net profit $Z_1(T)$ will also increase. Similarly if selling price 'p' increases, order quantity (Q_1) decreases while net profit $Z_1(T)$ increases. That is, change in 'α' leads positive change in (Q_1) and $Z_1(T)$ and the change in 'β' causes negative change in (Q_1) and positive change in $Z_1(T)$
- When the cash demand 'α' increases, order quantity (Q_1) and net profit $Z_1(T)$ will also increase. Similarly if ordering cost's' increases, order quantity (Q_1) increases while net profit $Z_1(T)$ decreases. That is, change in 'α' leads positive change in (Q_1) and negative change in $Z_1(T)$ and the change in 's' causes negative change in both (Q_1) and $Z_1(T)$

Table 1. Variation of cash demand 'α' and credit demand 'β'

α↓	β→	0.2	0.3	0.4	0.5	0.6	0.7
1000	T	0.225457	0.225541	0.225625	0.225709	0.225794	0.22588
	Q	226.620000	227.289000	227.960000	228.632000	229.308000	229.98600
	$Z_1(T)$	5869.550000	5895.810000	5922.130000	5948.520000	5974.980000	6001.50000
1100	T	0.215346	0.215433	0.215522	0.215610	0.215699	0.215788
	Q	238.099000	238.807000	239.520000	240.233000	240.950000	241.660000
	$Z_1(T)$	6696.760000	6725.550000	6754.410000	6783.340000	6812.340000	6841.420000
1200	T	0.260542	0.206633	0.206725	0.206817	0.206910	0.207003
	Q	248.485000	249.870000	250.622000	251.375000	252.133000	252.779000
	$Z_1(T)$	7534.010000	7565.310000	7596.690000	7628.150000	7660.130000	7691.310000
1300	T	0.206542	0.198881	0.198976	0.199072	0.199169	0.199265
	Q	259.744000	260.532000	261.322000	262.116000	262.903000	263.712000
	$Z_1(T)$	8380.040000	8413.850000	8447.740000	8481.710000	8515.770000	8549.920000
1400	T	0.191887	0.191985	0.192084	0.192183	0.192283	0.192383
	Q	270.012000	270.838000	271.669000	272.501000	273.338000	274.177000
	$Z_1(T)$	9233.860000	9270.150000	9306.540000	9343.020000	9379.590000	9416.260000
1500	T	0.185679	0.185799	0.185901	0.186004	0.186107	0.186211
	Q	279.936000	280.829000	281.697000	282.569000	283.444000	284.323000
	$Z_1(T)$	10094.600000	10133.400000	10172.300000	10211.200000	10250.300000	10289.500000

Table 2. Variation of cash demand 'α' and unit purchase cost 'c'

α↓	c→	45	40	35	30	25	20
1000	T	0.232056	0.239482	0.247801	0.257201	0.267939	0.280359
	Q	232.65500	240.101000	248.443000	257.869000	268.637000	281.091000
	$Z_1(T)$	10967.40000	16096.400000	21230.800000	26371.400000	31519.000000	36674.700000
1100	T	0.221564	0.228573	0.236423	0.245295	0.255428	0.267149
	Q	244.348000	252.079000	260.737000	270.523000	281.700000	294.629000
	$Z_1(T)$	12297.100000	17931.300000	23571.000000	29217.200000	34870.700000	40532.600000
1200	T	0.212427	0.219070	0.226511	0.234921	0.244527	0.255637
	Q	255.567000	263.560000	272.514000	282.634000	294.192000	307.561000
	$Z_1(T)$	13636.600000	19775.600000	25920.500000	32071.900000	38230.900000	44398.600000
1300	T	0.204376	0.210696	0.217775	0.225777	0.234917	0.245488
	Q	266.369000	274.607000	283.835000	294.266000	306.181000	319.961000
	$Z_1(T)$	14984.700000	21628.300000	28278.000000	34934.400000	41598.600000	48271.900000
1400	T	0.197213	0.203244	0.210000	0.217637	0.226361	0.236450
	Q	276.803000	285.270000	294.754000	305.475000	317.722000	331.886000
	$Z_1(T)$	16340.300000	23488.400000	30642.700000	37803.900000	44973.100000	52151.500000
1500	T	0.190786	0.196557	305.312000	0.210330	0.218679	0.228335
	Q	286.908000	295.588000	0.203022	316.304000	328.862000	343.386000
	$Z_1(T)$	17702.800000	25355.200000	33013.800000	40679.600000	48353.500000	56036.900000

Table 3. Variation of cash demand 'α' and unit selling price 'p'

α↓	p→	65	70	75	80	85	90
1000	T	0.221683	0.218147	0.214756	0.211500	0.208369	0.205356
	Q	222.254000	218.708000	215.308000	212.043000	208.903000	205.882000
	$Z_1(T)$	10794.800000	15747.300000	20701.000000	25655.700000	30611.500000	35568.200000
1100	T	0.211703	0.208297	0.205030	0.201892	0.198875	0.195971

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Table 3. Continue

	Q	233.471000	229.714000	226.110000	222.649000	219.321000	216.118000
	$Z_1(T)$	12118.400000	17570.000000	23022.800000	28476.700000	33931.700000	39387.700000
1200	T	0.203012	0.199717	0.196557	0.193521	0.190602	0.187792
	Q	244.238000	240.273000	236.470000	232.817000	229.305000	225.924000
	$Z_1(T)$	13452.300000	19403.100000	25355.200000	31308.400000	37262.900000	43218.400000
1300	T	0.195354	0.192157	0.189089	0.186142	0.183308	0.180580
	Q	254.608000	250.441000	246.441000	242.600000	238.905000	235.349000
	$Z_1(T)$	14795.100000	21245.300000	27696.900000	34149.600000	40603.700000	47058.900000
1400	T	0.188541	0.185429	0.182442	0.179574	0.176814	0.174158
	Q	264.629000	260.260000	256.067000	252.041000	248.166000	244.437000
	$Z_1(T)$	16145.90000	23095.700000	30046.800000	36999.800000	43953.100000	50908.100000
1500	T	0.182428	0.179391	0.176477	0.173678	0.170985	0.168392
	Q	274.336000	269.768000	265.385000	261.175000	257.125000	253.225000
	$Z_1(T)$	17503.80000	24953.200000	32404.100000	39856.400000	47310.100000	54765.100000

Table 4. Variation of cash demand 'α' and ordering cost's'

$\alpha\downarrow$	$s\rightarrow$	550	600	650	700	750	800
1000	T	0.236001	0.246173	0.255946	0.265364	0.274464	0.283279
	Q	236.611000	246.811000	256.611000	266.054000	275.179000	284.019000
	$Z_1(T)$	5614.060000	5394.830000	5184.460000	4981.960000	4786.500000	4597.390000
1100	T	0.225374	0.235055	0.244355	0.253318	0.261978	0.270365
	Q	248.550000	257.519000	269.487000	279.373000	288.925000	298.176000
	$Z_1(T)$	6427.700000	6197.910000	5977.400000	5765.120000	5560.220000	5361.970000
1200	T	0.216120	0.225374	0.234263	0.242830	0.251107	0.259123
	Q	260.010000	271.146000	281.842000	292.150000	302.110000	311.756000
	$Z_1(T)$	7251.910000	7012.040000	6781.840000	6560.240000	6346.330000	6134.360000
1300	T	0.207968	0.216846	0.225374	0.233591	0.241531	0.249221
	Q	271.051000	282.624000	293.741000	304.453000	314.803000	324.827000
	$Z_1(T)$	8085.360000	7835.840000	7596.370000	7365.840000	7143.290000	6927.970000
1400	T	0.200716	0.209259	0.217466	0.225374	0.233014	0.240413
	Q	281.721000	293.714000	305.235000	316.337000	327.062000	337.449000
	$Z_1(T)$	8927.00000	8668.210000	8419.830000	8180.710000	7949.880000	7726.520000
1500	T	0.19421	0.202453	0.210372	0.218002	0.225374	0.232512
	Q	292.05800	304.456000	316.367000	327.844000	338.932000	349.668000
	$Z_1(T)$	9775.96000	9508.230000	9251.270000	9003.880000	8765.050000	8533.960000

Sensitivity Analysis (Case II)

Table 5. Variation of cash demand 'α' and credit demand 'β'. (at s = 15)

$\alpha\downarrow$	$\beta\rightarrow$	0.2	0.3	0.4	0.5	0.6	0.7
1000	T	0.0733754	0.0736225	0.0738738	0.0741293	0.0743888	0.0746524
	Q	73.5391000	73.7869000	74.0389000	74.2951000	74.5553000	74.8196000
	$Z_2(T)$	9523.0200000	9543.9600000	9564.9500000	9585.9800000	9607.0700000	9628.2100000
1100	T	0.0699907	0.0702495	0.0705127	0.0707801	0.0710517	0.0713275
	Q	77.1596000	77.4451000	77.7354000	78.0303000	78.3299000	78.6341000
	$Z_2(T)$	10523.9000000	10546.5000000	10569.2000000	10592.0000000	10614.8000000	10637.7000000
1200	T	0.0670415	0.0673116	0.0675860	0.0678648	0.0681479	0.0684352
	Q	80.6254000	80.9504000	81.2806000	81.6160000	81.9567000	82.3024000
	$Z_2(T)$	11526.7000000	11551.0000000	11575.4000000	11599.9000000	11624.4000000	11649.0000000
1300	T	0.0644423	0.0647230	0.0650082	0.0652979	0.0655919	0.0658902
	Q	83.9559000	84.3218000	84.6936000	85.0712000	85.4545000	85.84330000
	$Z_1(T)$	12531.2000000	12557.2000000	12583.2000000	12609.3000000	12635.5000000	12661.7000000
1400	T	0.0621289	0.0624200	0.0627155	0.0630156	0.0633200	0.0636288
	Q	87.1664000	87.5751000	87.9899000	88.4112000	88.8385000	89.2720000
	$Z_2(T)$	13537.3000000	13564.8000000	13592.4000000	13620.2000000	13648.0000000	13675.8000000
1500	T	0.0600530	0.0603539	0.0606594	0.0609694	0.0612839	0.0616028
	Q	90.2702000	90.7288000	91.1823000	91.6486000	92.1216000	92.60130000
	$Z_2(T)$	14544.7000000	14573.8000000	14603.0000000	14632.2000000	14661.6000000	14691.1000000

Table 6. Variation of cash demand 'α' and unit purchase cost 'c' (at s = 15)

α↓	c→	45	40	35	30	25	20
1000	T	0.0692223	0.0659650	0.0632006	0.0608191	0.0587420	0.0569111
	Q	69.3746000	66.1084000	63.3364000	60.9483000	58.8655000	57.0296000
	$Z_2(T)$	14567.7000000	19624.6000000	24675.1000000	29720.6000000	34762.2000000	39800.5000000
1100	T	0.0660548	0.0629902	0.0603911	0.0581534	0.0562058	0.0544845
	Q	72.8182000	69.4379000	66.5711000	64.1028000	61.9513000	60.0560000
	$Z_2(T)$	16070.0000000	21629.9000000	27183.1000000	32731.1000000	38274.9000000	43815.5000000
1200	T	0.0632960	0.0604009	0.0579471	0.0558359	0.0536203	0.0523774
	Q	76.0342000	72.6348000	69.6821000	67.1418000	64.4757000	62.9802000
	$Z_2(T)$	17574.2000000	23636.9000000	29692.6000000	35743.0000000	41790.8000000	47831.8000000
1300	T	0.0608652	0.0581210	0.0557966	0.0537979	0.0520578	0.0505267
	Q	79.2929000	75.7157000	72.6857000	70.8020000	67.8119000	65.8160000
	$Z_2(T)$	19080.1000000	25645.4000000	32203.6000000	38756.2000000	45304.5000000	51849.3000000
1400	T	0.0587027	0.0560939	0.0538858	0.0519882	0.0503371	0.0488852
	Q	82.3566000	78.6942000	75.5944000	72.9305000	70.6126000	68.57440000
	$Z_2(T)$	20587.4000000	27655.2000000	34715.8000000	41770.6000000	48821.0000000	55867.7000000
1500	T	0.0567626	0.0542767	0.0521738	0.0503678	0.0487973	0.0474171
	Q	85.3211000	81.5820000	78.4190000	75.7025000	73.3403000	71.2643000
	$Z_2(T)$	22096.0000000	29666.2000000	37229.0000000	44786.0000000	52338.4000000	59887.1000000

Table 7. Variation of cash demand 'α' and unit selling price 'p' (at s =15)

α↓	p→	65	70	75	80	85	90
1000	T	0.0673476	0.0628486	0.0592261	0.0562323	0.0537072	0.0515423
	Q	67.4947000	62.9834000	59.3510000	56.3489000	53.8169000	51.6461000
	$Z_2(T)$	14554.6000000	19600.1000000	24641.0000000	29678.8000000	34714.5000000	39748.6000000
1100	T	0.0642605	0.0600045	0.0565795	0.0537505	0.0513657	0.0493221
	Q	70.8391000	66.1446000	62.3668000	59.2464000	56.6159000	54.3618000
	$Z_2(T)$	16058.2000000	21607.5000000	27152.7000000	32694.3000000	38233.6000000	43771.3000000
1200	T	0.0615714	0.0575284	0.0542767	0.0515922	0.0493305	0.0473934
	Q	74.0432000	69.1783000	65.2656000	62.0353000	59.3139000	56.9830000
	$Z_2(T)$	17563.8000000	23617.5000000	29666.2000000	35711.6000000	41754.6000000	47795.9000000
1300	T	0.0592019	0.0553478	0.0522498	0.0496936	0.0475411	0.0456986
	Q	77.1247000	72.1006000	68.0622000	64.7300000	61.9241000	59.5222000
	$Z_2(T)$	19071.1000000	25628.8000000	32181.5000000	38730.6000000	45277.2000000	51822.1000000
1400	T	0.0570936	0.0534087	0.0504484	0.0480071	0.0459524	0.0441946
	Q	80.0977000	74.9246000	70.7688000	67.3417000	64.4572000	61.9895000
	$Z_2(T)$	20579.9000000	27641.7000000	34698.200000	41751.0000000	48801.2000000	55849.6000000
1500	T	0.0552021	0.0516700	0.048834	0.0464965	0.0445302	0.0428488
	Q	82.9739000	77.6612000	73.395500	69.8797000	66.9221000	64.3931000
	$Z_2(T)$	22090.0000000	29655.8000000	37216.100000	44772.6000000	52326.5000000	59878.4000000

Table 8. Variation of cash demand 'α' and ordering cost's'

α↓	s→	14	13	12	11	10	5
1000	T	0.0706576	0.06809410	0.0654316	0.0626576	0.0597567	0.0424105
	Q	70.8138000	68.24330000	65.5735000	62.7919000	59.8830000	42.4892000
	$Z_2(T)$	9534.5200000	9568.08000000	9602.9200000	9639.2100000	9677.1400000	9903.4800000
1100	T	0.0673782	0.06493600	0.0623994	0.0597567	0.0569931	0.0404705
	Q	74.2780000	71.58420000	68.7863000	65.8713000	62.8230000	44.5983000
	$Z_2(T)$	10535.2000000	10570.30000000	10606.8000000	10665.2000000	10684.6000000	10921.6000000
1200	T	0.0645200	0.06218340	0.0597567	0.0572284	0.0545845	0.0387804
	Q	77.5913000	74.77970000	71.8596000	68.8173000	65.6360000	46.6190000
	$Z_2(T)$	11537.8000000	11574.50000000	11612.6000000	11652.2000000	11693.7000000	11940.8000000
1300	T	0.0620001	0.05975670	0.0574268	0.0549995	0.0524612	0.0372910
	Q	80.7723000	77.84790000	74.8107000	71.6466000	68.3377000	48.5624000
	$Z_2(T)$	12542.1000000	12580.30000000	12619.9000000	12661.1000000	12704.2000000	12961.0000000
1400	T	0.0597567	0.05759963	0.0553527	0.0530152	0.0505710	0.0359657
	Q	83.8362000	80.80340000	77.6537000	74.3722000	70.9410000	50.4374000
	$Z_2(T)$	13548.0000000	13587.50000000	13628.6000000	13671.8000000	13716.0000000	13982.0000000
1500	T	0.0597567	0.05565700	0.0534908	0.0512341	0.0488744	0.0347766
	Q	83.8362000	83.65810000	80.3999000	77.0055000	73.4563000	52.2516000
	$Z_2(T)$	14532.4000000	14596.10000000	14638.5000000	14682.7000000	14728.9000000	15003.8000000

Conclusion and Future Research

In this study, the retailer's optimal ordering policy under two stage trade credit financing is developed using Discounted Cash Flow (DCF) approach. An algorithm is established to obtain the optimal solution. The sensitivity analysis of the optimal solution with respect to the parameters is also discussed. The results show some phenomena as follows: (i). A higher value of cash demand 'α' caused higher value of retailer's annual profit, (ii). A higher value of credit demand 'β' causes higher value of retailer's annual profit, (iii). A higher value of unit purchase cost 'c' causes lower value of retailer's annual profit, (iv). A higher value of selling price 'p' causes higher value of net profit. That is, the retailer should increase the net profit per transfer from the increase of cash demand, credit demand, purchase cost and selling price. Second order approximation is used for exponential terms to find exact values of cycle time 'T', order quantity 'Q' and retailer's annual profit $Z(T)$.

The proposed model can be further extended in several ways. For example, we may add pricing strategy into consideration. We may also extend the model to allow for constant deterioration rate or a two-parameter weibull distribution. In addition, we can consider the demand as a function of time, price as well as quality. Finally, we could generalize the model to allow for shortages, quantity discount or others.

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Harenrda Singh: Coordination of research work and publication of the manuscript.

Neha Sang: Design the research plan, organization, development and publication of the manuscript.

Ethics

In this paper second order approximation have been used for exponential terms to find closed form optimal solution. With the help of differential calculus the author's have obtained the retailer's annual net profit per unit time.

Appendix A

To prove this appendix, we first prove the following lemma.

Lemma 1: If a function $G(T)=\frac{F(T)}{T}$, where F(T) is a differential function of T two times, then the maximum value of $G(T)$ exist at $T = T^*$ if $\frac{1}{T}\frac{d^2(F(T))}{dT^2}<0$, at $T = T^*$.

Proof: It is given that $G(T)=\frac{F(T)}{T}$. For extremum, the necessary condition is $\frac{d(G(T))}{dT}=0$. But

$$\frac{d(G(T))}{dT}=-\frac{F(T)}{T^2}+\frac{1}{T}\frac{d(F(T))}{dT}=\frac{1}{T^2}\left(T\frac{d(F(T))}{dT}-F(T)\right)$$

$\frac{d(G(T))}{dT}=0$, gives $T\frac{d(F(T))}{dT}-F(T)=0$. (i)

Let Equation (i) be satisfied for $T = T^*$.

Again

$$\frac{d^2(G(T))}{dT^2}=-\frac{2}{T^3}\left(T\frac{d(F(T))}{dT}-F(T)\right)+\frac{1}{T}\frac{d^2(F(T))}{dT^2}$$

Or $\frac{d^2(G(T))}{dT^2}=\frac{1}{T}\frac{d^2(F(T))}{dT^2}$ from (i)

We know that the sufficient condition for existence of a maximum value of $G(T)$ is $\frac{d^2(G(T))}{dT^2}<0$. Hence the Lemma 1.

Here $Z_1(T)=\frac{1}{T}[SR+IE_1-PC-OC-IC-IP]$. For extremum value at $T = T^*$

Where Sales revenue = SR/T, interest earned = IE_1/T, Purchase cost = PC/T. Ordering cost = OC/T, Inventory carrying cost = IC/T, Interest payable = IP/T.

At $T = T^*$, the necessary condition is $\frac{dZ_1(T)}{dT}=0.$, which gives Equation (18).

If $T = T^*$ be a maximum value of $Z_1(T)$, then at $T = T^*$, we have

$$\frac{d^2Z_1(T)}{dT^2}=\frac{1}{T}\left[\begin{array}{l}\frac{d^2(SR)}{dT^2}+\frac{d^2(IE_1)}{dT^2}-\frac{d^2(PC)}{dT^2}\\-\frac{d^2(OC)}{dT^2}-\frac{d^2(IC)}{dT^2}-\frac{d^2(IP)}{dT^2}\end{array}\right].\text{ By Lemma 1.}$$

Now at $T = T^*$

$$\frac{d^2Z_1(T)}{dT^2}=-\frac{re^{-rT}}{T}\left(rp+ic+cI_p\right)<0.$$

Hence the proof of Appendix A.

Appendix B:

We have $Z_2(T) = \frac{1}{T}\left[SR + IE_2 - PC - OC - IC\right]$.

Where Sales revenue = *SR/T*, interest earned = IE_2/T, Purchase cost = *PC/T* . Ordering cost = *OC/T*, Inventory carrying cost = *IC/T*.

For extremum value of $Z_2(T)$, the necessary condition is $\frac{dZ_2(T)}{dT} = 0$, which gives Equation (19). If T = T** be a maximum value of Z_2(T), then at $T = T^{**}$ $\frac{d^2Z_2(T)}{dT^2} = \frac{1}{T}\left[\frac{d^2(SR)}{dT^2} + \frac{d^2(IE_2)}{dT^2} - \frac{d^2(PC)}{dT^2} - \frac{d^2(OC)}{dT^2} - \frac{d^2(IC)}{dT^2}\right]$.

by Lemma 1. Now at $T = T^{**}$ $\frac{d^2Z_2(T)}{dT^2} = -\frac{re^{-rT}}{T}\left\{rp\left(1 + \frac{I_e}{r}\right) + ic\right\} < 0.$

Hence the proof of Appendix B.

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