

## Viscous Dissipation Effect on Steady free Convection and Mass Transfer Flow past a Semi-Infinite Flat Plate

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**Abstract: Problem statement:** A steady two-dimensional free convection and mass transfer flow past a continuously moving semi-infinite flat plate is studied theoretically, by taking into account the viscous dissipation effect. **Approach:** The governing equations are transformed into a set of simultaneous ordinary differential equation by using suitable similarity transformations. The coupled differential equations are integrated using the Runge-Kutta Gill method together with the shooting technique. **Results:** Numerical results were presented for the distribution of velocity, temperature and concentration profiles within the boundary layer. **Conclusion:** The effects of varying parameter  $G_b$ , the Gebhart number,  $Sc$ , Schmidt number and  $Pr$ , Prandtl number on the velocity, temperature and concentration profiles were displayed graphically for different values of parameters entering into the problem. Significant changes were observed in heat and mass transfer coefficient, due to viscous dissipation in the medium. In addition, the skin-friction coefficient, Nusselt number and Sherwood number were shown in a tabular form.

**Key words:** Heat transfer, mass transfer, moving surface, prandtl number, gebhart number, schmidt number, viscous dissipation, skin-friction coefficient, sherwood number

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### INTRODUCTION

Owing to their numerous applications in industrial manufacturing process, the problem of heat and mass transfer in the boundary layers of a continuously moving semi-infinite flat plate has attracted the attention of researchers for the past 3 decades. Some of the application areas are hot rolling, study production, metal spinning, drawing plastic films, glass blowing, continuous casting of metals and spinning of fibers. Annealing and thinning of copper wire is another example. In all these cases, the quality of the final product depends on the rate of heat and mass transfer at the moving surface. By drawing the strips in an electrically conducting fluid subjected to a magnetic field the rate of cooling can be controlled and the final products of desired characteristic might be achieved.

Flow in the boundary layer of a viscous fluid on a moving continuous solid surfaces was investigated by Sakiadis (1961a). It was observed that the boundary layer growth is in the direction of motion of the continuous solid surface and is different from that of the Blasius flow past a flat plate. Still the boundary

layer theory and the basic differential equations are applicable. Many authors have attacked this problem, however, this attack has been limited to some constrains on the surface velocity and temperature distribution. In addition to these investigations, experimental and theoretical studies of the flow and temperature fields in the boundary layer on a continuous moving surface have been made by Tsou *et al.* (1967) for different values of the Prandtl number. Measure of the laminar velocity field were in excellent agreement with the analytical predictions. The heat transfer of above problem of different physical situations have been studied. Viscous dissipation which, appears as a source term in the fluid flow generates appreciable temperature, gives the rate at which mechanical energy is converted into heat in a viscous fluid per unit volume. This effect is of particular significant in natural convection in various devices that are subjected to large variation of gravitational force or that operate at high rotational speeds, pointed by Gebhart (1962) in his study of viscous dissipation on natural convection in fluids. Similarity solutions for the same problem with exponential variation of wall

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temperature was obtained by Gebhart and Mollendorf (1969). It is observed that the effect of viscous dissipation is more predominant in vigorous natural convection and mixed convection processes. They also have shown the existence of similarity solution for the external flow over an infinite flat vertical surface having an exponential variation of surface temperature. Vajravelu and Hadjinicolaou (1993) studied the heat transfer characteristic in the laminar boundary layer of a viscous fluid over a linearly stretching surface with variable wall temperature subject to suction or blowing. They considered the effects of viscous dissipation and internal heat generation. The effect of chemical reaction, heat and mass transfer on a laminar flow along a semi infinite horizontal plate have been studied by Anjali and Kandasamy (1999). Studied the effect of viscous dissipation on heat transfer in flow past a continuously moving semi-infinite flat plate. Natural convection boundary layer flow over a sphere of a viscous incompressible electrically conducting fluid in the presence of magnetic field and heat generation with the effects of viscous dissipation has been investigated by Alam *et al.* (2007) and Subhas Abel *et al.* (2011) studied the effect of viscous and Joules dissipation on MHD flow over a porous nonlinear vertical stretching sheet with partial slip. Since the pioneering work of Sakiadis (1961b) which studied the moving plate flow problem, wherein various aspects of the problem have been investigated by Cortell (2006). Cortell (2007a) has worked on viscous flow and heat transfer over a non-linearly stretching sheet. Cortell (2008) further investigated on the effects of viscous dissipation and radiation on the thermal boundary layer, over a non-linear stretching sheet. Moreover, Cortell (2007b) studied the viscous flow and heat transfer over a non-linearly stretching surface. Raptis and Perdakis (2006) studied viscous flow near a non-linear stretching sheet in the presence of a chemical reaction and magnetic field. Kumar (2009) investigated the study of radiation and viscous dissipation effects over a stretching surface subjected to variable heat flux in presence of transverse magnetic field. Kishan and Amrutha (2010) studied the two-dimensional steady nonlinear MHD boundary layer flow of an incompressible, viscous, electrically conductive and Boussinesq fluid flowing over a vertical stretching surface in the presence of uniform magnetic field by taking into account the viscous dissipation with heat, mass transfer chemical reaction and thermal stratification effects. A study on MHD free convective flow of an incompressible viscous dissipative fluid in

an infinite vertical oscillating plate with constant heat flux has been carried by Kishan *et al.* (2006). Kairi *et al.* (2011) investigated the combined effect of viscous dissipation and radiation on natural convection in non-Darcy porous medium saturated with non-Newtonian fluid of variable viscosity. El-Arabawy (2009) studied the effects of suction/injection and chemical reaction on mass transfer over a stretching surface. Adegbe and Alao (2007). investigated the steady-state flow of Newtonian liquid with exponential temperature-dependent viscosity and substantial viscous heat generation between symmetrically parallel heated walls with walls at different temperatures.

The aim of the present study is to study the effect of viscous dissipation on heat transfer and mass transfer in flow past a continuously moving semi-infinite flat plate. The analysis showed that the viscous dissipation have significant influence on the non-dimensional heat and mass transfer coefficients.

**Mathematical formulation:** Consider the two-dimensional, flow past a continuously moving semi-infinite flat plate. The x-axis is assumed to be taken along the plate and the y-axis normal to the plate.  $u, v$  are the velocity components in the x and y directions. Now, the governing equations under the boundary layer and Boussinesq approximations may be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \tag{4}$$

Along with the boundary conditions:

$$\begin{aligned} u = U_0; \quad v = 0; \quad T = T_w; \quad C = C_w \quad \text{at } y = 0 \\ u = 0; \quad T \rightarrow T_\infty; \quad C \rightarrow C_\infty \quad \text{at } y \rightarrow \infty \end{aligned} \tag{5}$$

Here:

$a$  = Thermal diffusivity constant

$c_p$  = Specific heat  
 $\nu$  = Kinematic viscosity of the ambient fluid

The above set of partial differential equations are converted into ordinary differential equations using the following similarity transformation:

$$\eta = y \sqrt{\frac{U_0}{\nu x}} \tag{6}$$

$$\psi(x, y) = \sqrt{\nu x U_0} f \tag{7}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{9}$$

Where:

$\eta$  = The similarity variable  
 $\theta$  = The dimensionless stream function depends on  $\eta$  only

Let  $\psi$  be the stream function defined such that  
 $u = \frac{\partial \psi}{\partial y}$  and  $v = -\frac{\partial \psi}{\partial x}$  so that it automatically satisfies the continuity Eq. 1. The final transformed equation and boundary conditions are:

$$f'' + \frac{1}{2} f f'' = 0 \tag{10}$$

$$\theta'' + \frac{1}{2} Pr f \theta' + Gb Pr (f'')^2 = 0 \tag{11}$$

$$\phi'' + \frac{Sc}{2} f \phi' = 0 \tag{12}$$

With the boundary and initial conditions as:

$$\begin{aligned} f(0) = 0; f'(0) = 1; \theta(0) = 1; \phi(0) = 1 \quad \text{at } \eta = 0 \\ f(\infty) = 0; \theta(\infty) = 0; \phi(\infty) = 0 \quad \text{at } \eta \rightarrow \infty \end{aligned} \tag{13}$$

The non-dimensional numbers are defined as the viscous dissipation parameter known as the Gebhart number, given by  $Gb = \frac{U^2}{c_p(T_w - T_\infty)}$ , Schmidt number given by  $Sc = \nu/D$  and the Prandtl number given by  $Pr = \nu/a$ .

### MATERIALS AND METHODS

The set of Eq. 10-12 together with the boundary conditions (11) have been solved numerically by applying shooting technique along with Runge-Kutta Gill method. From the process of numerical computation, the skin-friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to  $f''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$  are also sorted out and their numerical values are presented in a tabular form.

The parameters for the present problem are the local Nusselt number and the local Sherwood number, which indicate physically the rate of heat transfer and the rate of mass transfer respectively.

Now the heat flux ( $q_w$ ) and the mass flux ( $M_w$ ) at the wall are given by:

$$q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} = -k(T_w - T_\infty) \sqrt{\frac{U_0}{\nu x}} \theta'(0)$$

And:

$$M_w = -D \left( \frac{\partial C}{\partial y} \right)_{y=0} = -D(C_w - C_\infty) \sqrt{\frac{U_0}{\nu x}} \phi'(0)$$

Hence the Nusselt number (Nu) and Sherwood number (Sh) are obtained as:

$$Nu = \frac{x q_w}{k(T_w - T_\infty)} = -(Re)^{\frac{1}{2}} \theta'(0)$$

i.e.  $Nu (Re)^{-\frac{1}{2}} = -\theta'(0)$  and

$$Sh = \frac{x M_w}{D(C_w - C_\infty)} = -(Re)^{\frac{1}{2}} \phi'(0)$$

i.e.  $Sh (Re)^{-\frac{1}{2}} = -\phi'(0)$

where,  $Re = U_0 x / \nu$  is the Reynold's number.

The ordinary differential Eq. 10-12 along the boundary condition Eq. 13 are solved by giving approximate initial guess values for the missing initial conditions of  $f'(0)$ ,  $\theta'(0)$ ,  $\phi'(0)$  and these values are matched with the corresponding boundary conditions at  $f'(\infty)$ ,  $\theta(\infty)$  and  $\phi(\infty)$ . Extensive calculations have been

performed to obtain the flow and temperature fields for a wide range of parameters  $0 < Pr \leq 10$ ,  $0 < Sc \leq 1$  and  $1 \leq Gb \leq 10$ .

**RESULTS**

Table 1 shows numerical values of viscous dissipation effects on  $f''(0)$ ,  $\theta'(0)$  and  $\phi'(0)$ .

The profiles for velocity, temperature and concentration are shown in Fig. 1-7 respectively with various values of the parameters.

Table 1: Viscous dissipation effects on  $f''(0)$ ,  $\theta'(0)$  and  $\phi'(0)$

| Sc   | Pr   | Gb | $f''(0)$ | $\theta'(0)$ | $\phi'(0)$ |
|------|------|----|----------|--------------|------------|
| 0.3  | 0.71 | 1  | -0.4439  | -0.1894      | -0.2183    |
|      |      | 5  | -0.4439  | 0.4710       | -0.2490    |
|      |      | 10 | -0.4439  | 1.2966       | -0.2873    |
| 0.6  | 7    | 1  | -0.4485  | -0.2068      | -0.3268    |
|      |      | 5  | -0.4485  | 4.0156       | -0.5455    |
|      |      | 10 | -0.4485  | 9.4169       | -0.8166    |
| 0.78 | 10   | 1  | -0.4539  | -0.2270      | -0.3910    |
|      |      | 5  | -0.4539  | 5.5742       | -0.6939    |
|      |      | 10 | -0.4539  | 12.8260      | -1.0723    |

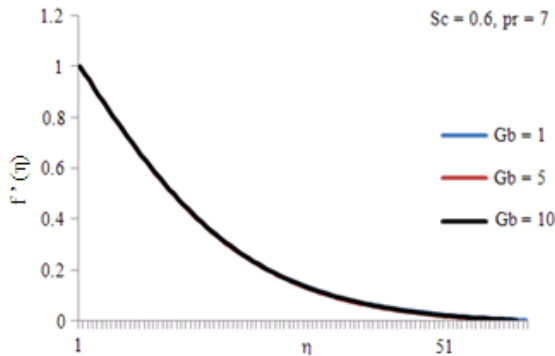


Fig. 1: Effect of viscous dissipation parameter Gb on non-dimensional velocity  $f'$

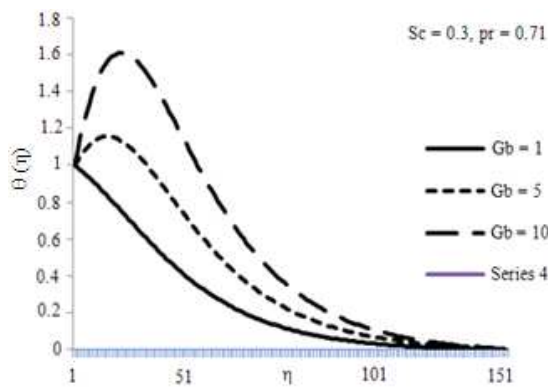


Fig. 2: Effect of viscous dissipation parameter Gb on non-dimensional temperature  $\theta$

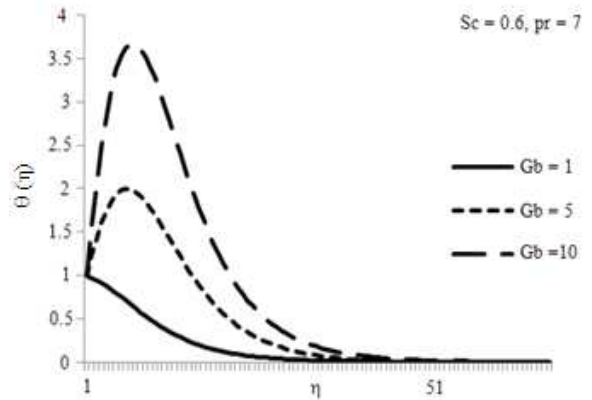


Fig. 3: Effect of viscous dissipation par  $\theta$

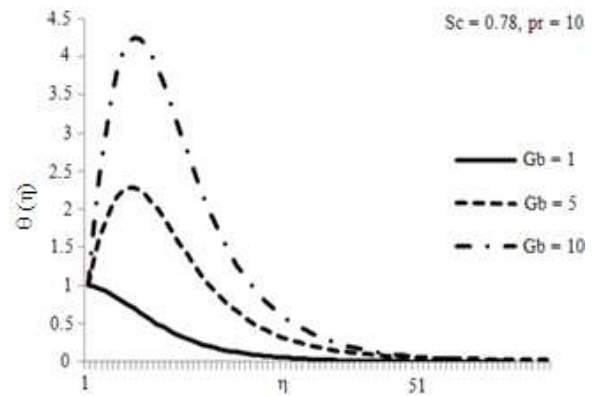


Fig. 4: Effect of viscous dissipation parameter Gb on non-dimensional temperature  $\theta$

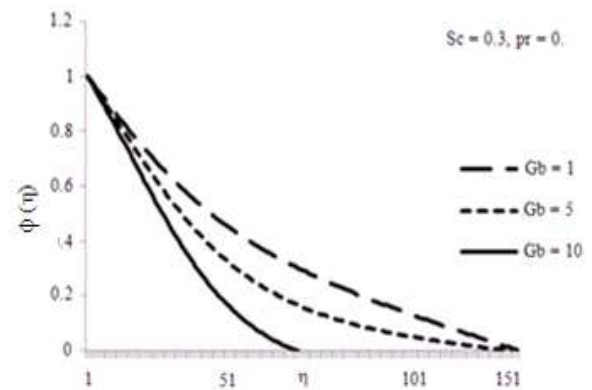


Fig. 5: Effect of viscous dissipation parameter Gb on non-dimensional concentration  $\phi$

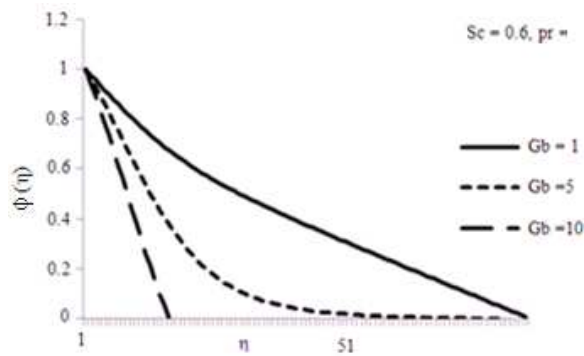


Fig. 6: Effect of viscous dissipation parameter  $G_b$  on non-dimensional concentration  $\phi$

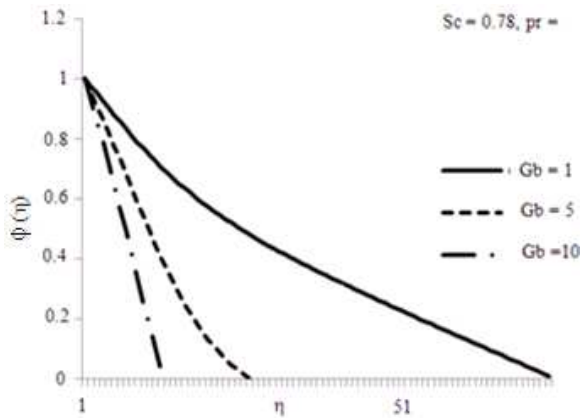


Fig. 7: Effect of viscous dissipation parameter  $G_b$  on non-dimensional concentration  $\phi$

### DISCUSSION

Now we discuss the results. The velocity, temperature and concentration profiles in the boundary layer are plotted for selected values of the governing parameters. The non-dimensional velocity across the boundary layer is plotted for varying values of  $G_b$  (1,5,10) with Prandtl number  $Pr = 0.71, 7, 10$  and Schmidt number  $Sc = 0.3, 0.6, 0.78$  to depict the flow field. The magnitude of the velocity remains same as the value of the flow governing parameter increases which is shown in Fig. 1.

The non-dimensional temperature distribution inside the boundary layer are presented in Fig. 2-4. The thermal boundary layer thickness is increased with the increasing of  $G_b$ .

The non-dimensional concentration distribution inside the boundary layer are presented in Fig. 5-7. The concentration decreases with the increase of  $G_b$ .

The non-dimensional heat transfer coefficient and mass transfer coefficient are plotted against the parameter  $G_b$ . The heat transfer coefficient and mass transfer coefficient are increased with the Prandtl number and also as the value of the flow governing parameter is increased.

Table 1 presents the effect of  $Sc, Pr$  and  $G_b$  on the skin friction coefficient  $f''(0)$ , the Nusselt number  $-\theta'(0)$  and the Sherwood number  $-\phi'(0)$ . The results reveal that the Nusselt number and the Sherwood number increases as  $Sc, Pr$  and  $G_b$  increases.

### CONCLUSION

An increase in the viscous dissipation parameter  $G_b$  resulted:

- An increase in temperature
- A fall in concentration
- An increase in heat transfer coefficient and mass transfer coefficient

### ACKNOWLEDGEMENT

The research render their heartfelt thanks to Dr. T. Govindarajulu former Professor and Head of Department of Mathematics, Anna University, Chennai for his generous help. The authors wish to thank the Director IRT Chennai and the Principal IRTT, Erode for their generous help. The authors wish to thank the Chairman, Director, Chief Executive, Principal, Bannari Amman Institute of Technology for their support to do this study.

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