Energy Research Journal

Investigations

Comparison of Green Roof Model Predictions with Experimental Data

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Article history
Received: 20-09-2014
Revised: 07-11-2014
Accepted: 10-07-2015

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Abstract: In this study, the results of a green roof model are presented and compared with experimental data. Good agreement between simulation and experiment were found for different experimental conditions that were carried out in a controlled environment laboratory. The study of green roof heat transfer processes is important because it allows a better thermal design in buildings; the use of urban forest area reduces the urban heat island effect and others improvements such as air purification. Also, with a mathematical model that accurately describes the heat transfer in the green roof it is possible to determine variables such as the local and averaged temperature inside the building and thus obtain the energy savings of a green roof over a conventional one. In this study a mathematical model of heat transfer in a green roof considers that porous materials form some of its layers and that the heat sources are introduced into the model through boundary conditions. In order to validate the mathematical model considering porous materials, the data obtained from the model and laboratory experimental data were compared. The results show that the inclusion of equations considering porous materials, inside the heat transfer model of the green roof, described properly heat transfer processes. Considering porous materials in the green roof model allows including effects due to canopy density, the amount of water contained in the soil layer or penetration of the radiation through the green layer.

Keywords: Green Roof, Heat Transfer, Porous Media, Experimental Data, Mathematical Model

Introduction

Many studies about Green Roofs System (GRS) have been made in recent years due to the multiple benefits that they offer, such as increase in urban forest area (Berndtsson *et al*., 2009), air purification (Yang *et al*., 2008; Li *et al*., 2010), reducing runoff Mentens *et al*., 2006; Berndtsson *et al*., 2009; Stovin *et al*., 2012; Speak *et al*., 2013), reduction of the urban heat island effect (e.g., Wong *et al*., 2003; Weng and Yang, 2004; Wong and Chen, 2005; Takebayashi and Moriyama, 2007; Lin *et al*., 2008; Santamouris, 2012; Wong and Lau, 2013), life extension of the roof (e.g., Saiz *et al*., 2006; Kosareo and Ries, 2007; Clark *et al*., 2008), reducing the acoustical noise (e.g., Renterghem and Botteldooren, 2009), preservation of biodiversity (e.g., Madre *et al*., 2013) and energy saving in the building (e.g., Onmura *et al*., 2001; Theodosiou, 2003; Castleton *et al*., 2010; Zinzi and Agnoli, 2012, Hong *et al*., 2012; Chan and Chow, 2013).

Benefits, such as energy saving in the building and reducing the effect of urban heat island are examples of how the GRS alters the balance of energy in the roof (Sailor and Hagos, 2012). In order to quantify the energy savings in the building, equations are needed to describing accurately the heat transfer through the different layers comprising at GRS.

Early efforts of modeling GRS represented the green roof as a simple resistive layer whose thermal conductivity was essentially constant (Niachou *et al*., 2001). In recent years, sophisticated models for the energy balance in GRS have been developed (Sailor, 2008). Other models are: The model proposed by Del Barrio (1998), the model of Ayata *et al*. (2011), which considers four terms in the energy balance; heat gain due to net radiation, the sensible heat flux, the solid heat flux and the latent heat flux. And the model proposed by Kumar and Kaushik (2005).

Sailor (2008) proposed a more complete model, however, this model does not take into account the metabolic processes of plants such as photosynthesis and respiration, to name a few. The model proposed by Feng *et al.* (2010) includes in the energy balance the effects due to photosynthesis and plant respiration.

In previous works the heterogeneous effects of the green roof are not explicitly considered. In this study the heat transfer process in the green roof was consider as energy transport in porous media. The modeling results for heat transfer in a green roof are presented and compared with experimental data. Good agreement between simulation and experiment were found for different experimental conditions that were carried out in a controlled laboratory environment.

Mathematical Model

The energy balance model for GRS proposed by Feng *et al.* (2010) is based on the following assumptions:

- The lawn, with 100% leaf coverage, is considered as a diffuse gray body
- Thermal effects of plants metabolism except for photosynthesis, respiration and transpiration and thermal effects of microorganism in the soil are negligible
- The conditions with precipitation and dew are not included
- The GRS is large enough to assume horizontal homogeneity and apply a one-dimensional (vertical) analysis

Considering plants and soil as the system, structural roof and ambient air as the environment, the energy exchanges between the plants-soil system and the environment are obtained and illustrated in Fig. 1. According to the first law of thermodynamics, the following energy balance equation is obtained Equation 1:

$$q_{sr} + q_{lr} + q_{cv} + q_{em} + q_{tp} + q_{ep} + q_{sp} + q_{ss} + q_{tr} + q_{ps} + q_{rp} = 0 \qquad (1)$$

where, q_{sr} is the heat gain from solar radiation, q_{ly} the heat gain from long-wave radiation, q_{cv} the heat transferred by convection, q_{em} the heat loss by emission, q_{tp} the heat loss by transpiration, q_{ep} the heat loss by evaporation, q_{sp} the heat storage by plants, q_{ss} the heat storage by soil, q_{tr} the heat transferred into the room, q_{ps} the solar energy converted by photosynthesis and q_{rp} the heat generation by respiration.

The heat gain from solar radiation in the GRS is calculated by (Jim and Tsang, 2011) Equation 2:

$$q_{sr} = q_{sri}\alpha_s \qquad (2)$$

where, q_{sri} is the incident solar radiation and α_s is the short-wave absorptivity of the lawn.

The heat gain from long-wave radiation is calculated by (Meng and Hu, 2005) Equation 3:

$$q_{lr} = \alpha_l \sigma \left(T_a + 273.15\right)^4 \left(0.802 + 0.004 T_d\right) \qquad (3)$$

where, α_l is the long-wave absorptivity of the lawn, α is Stefan-Boltzmann constant, T_a is ambient air temperature in °C and T_d is dew point.

Many authors have dedicated their studies to propose a suitable method for the calculation of heat transferred by convection. Some authors have used Newton's law of cooling with minor modifications (e.g., Meng and Hu, 2005; Denardo, 2003; Jensen *et al.*, 1990). Some other authors have used a term called Leaf Area Index (LAI) (e.g., Niachou *et al.*, 2001; Tabares-Velasco and Srebric; 2009; Deardorff, 1978; Ayata *et al.*, 2011) that is defined as the total, one-sided area of leaves per unit ground surface area.

In this study is used the method developed by (Feng *et al.*, 2010 Equation 4):

$$q_{cv} = \left(5.7 + 3.8\upsilon\right)\left(T_p - T_a\right) \qquad (4)$$

where, υ is wind speed and T_p is the plant temperature.

Heat loss by emission can be calculated by (Jim and Tsang, 2011) Equation 5:

$$q_{em} = \sigma\varepsilon\left(T_p + 273.15\right)^4 \qquad (5)$$

where, ε is the emissivity of the lawn.

The heat loss by transpiration and evaporation can be simplified in one equation, so the heat loss by evapotranspiration is given by (Feng *et al.*, 2010) Equation 6:

$$q_{et} = R_{et} l \qquad (6)$$

where, R_{et} is the evapotranspiration rate and l is latent heat of vaporization.

The heat storage by plants is given by (Feng *et al.*, 2010) Equation 7:

$$q_{sp} = \rho_p C p_p \frac{dT_p}{dt} \qquad (7)$$

where, ρ_p is the areal density of plants, C_{pp} is the specific heat of plants and t is the time.

Similarly, Feng *et al.* (2010) proposed calculating the heat storage by soil with Equation 8:

$$q_{ss} = \rho_s C p_s \frac{dT_s}{dt} \qquad (8)$$

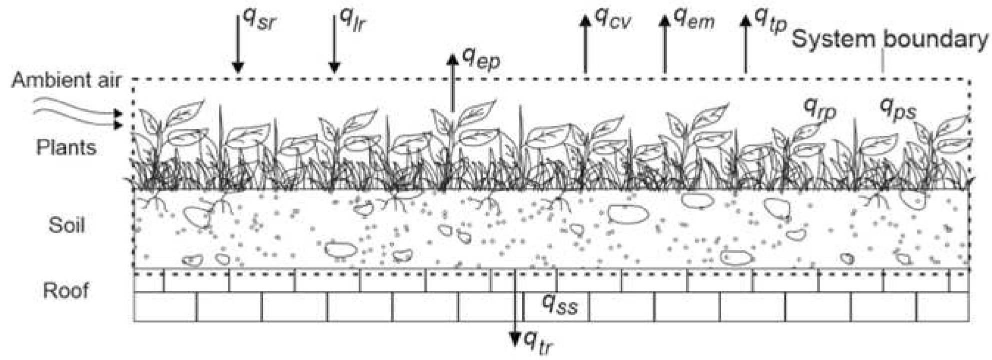

Fig. 1. Energy exchange a GRS and its environment (Feng *et al*., 2010)

The energy converted by photosynthesis and heat generation by respiration, if these two heat sources are added, is an equation used to determine the net heat converted by photosynthesis (Feng *et al*., 2010) Equation 9:

$$q_{ps,net} = \Delta_r H_m^{\theta}(25°C)\frac{\upsilon_{net},C_6H_{12}O_{16}}{M_{C_6H_{12}O_6}} \quad (9)$$

where, υ_{net}, $C_6H_{12}o_{16}$ is the net photosynthetic rate (in glucose), $\Delta_r H_m^{\theta}(25°C)$ is standard molar enthalpy of reaction at 25 °C and $M_{C6}H_{12}o_6$ is the glucose molar mass.

However the soil is a porous material and the composite layer by plants also can be considered a porous material because there is air between the plants. The Equation 9 and 10 in the Feng *et al*. (2010) do not take into account these considerations.

Porous Medium Approach

In this section the equations for porous media or two-phase media approach are presented.

The Fig. 2 illustrates how a material appears to be homogeneous on a large-scale and heterogeneous or a porous material on a smaller-scale. This is the case of the soil layer that is formed mainly by land and water. A similar case is given in the green layer that is composed by plants and air around them.

Starting from a local-instantaneous formulation, the equations for two-phase media can be derived. In the case of the soil layer the equations are given by Equation 10 and 11:

$$(\rho Cp)_{\ell}\frac{\partial T_{\ell}}{dt} = k_{\ell}\nabla^2 T_{\ell} + q_{\ell}''' \quad (10)$$

$$(\rho Cp)_{w}\frac{\partial T_{w}}{dt} = k_{w}\nabla^2 T_{w} + q_{w}''' \quad (11)$$

where, subscript ℓ refers to the land and the subscript *w* refers to water; q''' is heat source.

By an appropriate mathematical treatment it is possible to relate the Equation 10 and 11 for proposing an equation valid for a porous media with heat transfer (Espinosa-Paredes *et al*., 2013) Equation 12:

$$(\rho Cp)_{s}\frac{\partial\langle T\rangle}{\partial t} = \nabla\cdot\left(K_s\cdot\nabla\langle T\rangle\right) + \varepsilon_{\ell}\left\langle q'''\right\rangle^{\ell} \quad (12)$$

where, $\langle\ \rangle$ indicates average amounts, the subscript *s* indicate effective properties for the soil layer and ε_ℓ is the volume fraction of land in the soil layer. This equation is based on scale length restrictions, which is widely discussed in Espinosa-Paredes (2010; 2012).

The effective properties for the soil layer are given by Equation 13 and 14:

$$(\rho Cp)_{s} = \left(\rho_{\ell}Cp_{\ell}\varepsilon_{\ell} + \rho_{W}Cp_{w}\varepsilon_{w}\right) \quad (13)$$

$$K_s = \left(k_{\ell}\varepsilon_{\ell} + k_{w}\varepsilon_{w}\right) \quad (14)$$

It is important to note that these effective properties include the effect of the properties of land and water in the heat transfer through the soil layer. The effective properties are affected by the amount of land and water in the soil layer.

Similarly it is possible to obtain an equation valid for the green layer that considers plants and air:

$$(\rho Cp)_{g}\frac{\partial\langle T\rangle}{\partial t} = \nabla\cdot\left(K_g\cdot\nabla\langle T\rangle\right) + \varepsilon_{p}\left\langle q'''\right\rangle^{p} \quad (15)$$

where, the subscript *g* indicates the effective properties for the green layer and ε_p is the volume fraction of the plants. The full mathematical development for Equation 12 and 15 is presented in Appendix A.

In the next section it is presented the complete mathematical model for the heat transfer process through a green roof, GRS, which includes the effect of porous materials.

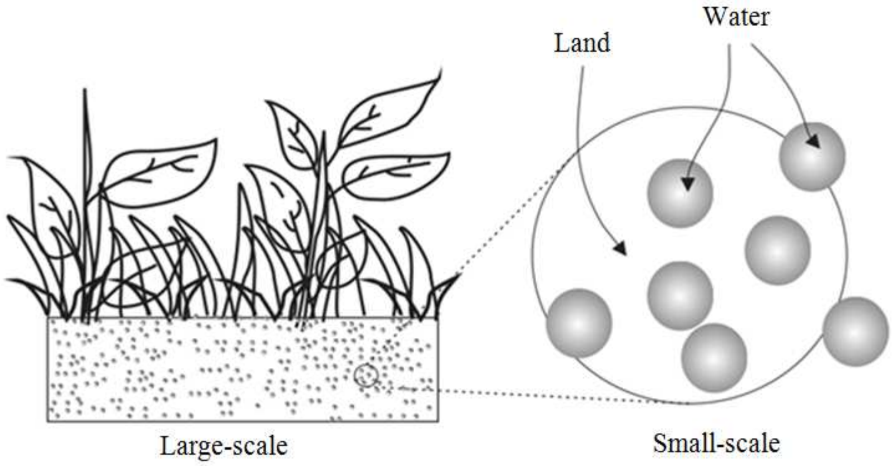

Fig. 2. Soil layer at different length-scales

Complete Mathematical Model

In this section it is presented the heat transfer model for green roofs considering that some of the component layers are porous materials.

The heat transfer in a green layer formed mainly by plants and air is given by:

$$(\rho Cp)_g \frac{\partial \langle T \rangle_g}{\partial t} = \nabla \cdot \left(K_g \cdot \nabla \langle T \rangle_g \right) \tag{16}$$

In Equation 16 are not included heat sources. A way to easily include the effect of heat sources is through the use of the following boundary condition that is valid in the interface formed by green layer and environment:

$$-K_g \frac{d\langle T \rangle_g}{dx} = \sum q'' \tag{17}$$

where, the term on the right side of the equation is given by Equation 18:

$$\sum q'' = q_{sr} + q_{lr} + q_{cv} + q_{em} + q_{et} + q_{ps,net} = 0 \tag{18}$$

The heat transfer in the soil layer formed mainly by land and water is given by Equation 12, without effect of heat sources that may be reaching the soil layer, such as solar radiation and long-wave radiation, these effects could be included by posing appropriate boundary condition.

Equation 12 and 16 are related by the following boundary condition and valid for all times:

$$\langle T \rangle_g = \langle T \rangle_s \tag{19}$$

The Equation 19 is valid on the interface formed by the green layer and the soil layer.

In addition the heat transfer through the building material is given by Equation 20:

$$(\rho Cp)_b \frac{\partial T_b}{\partial t} = k_b \frac{\partial^2 T_b}{\partial x^2} \tag{20}$$

where, subscript b indicates effective building properties.

At the interface formed by the construction material and the soil layer the following boundary condition is valid Equation 21:

$$T_b = \langle T \rangle_s \tag{21}$$

The boundary condition between the material construction element and the inside of building is given by Equation 22:

$$-k_b \frac{dT_b}{dx} = h(T_b - T_r) \tag{22}$$

where, T_r is the room temperature or the temperature inside the building.

The set of equations 16 to 22 represents a new model for the heat transfer process throughout the green roof.

Materials and Methods

In order to validate the inclusion of Equation 12 and 15 in the model to calculate the heat transport through the soil layer and through the green layer, an experimental model was designed. The Controlled Environment Laboratory (CEL) of the Universidad Autónoma Metropolitana-Iztapalapa (Arroyo-Cabañas *et al*., 2009), was used for monitoring several experimental scenarios of a green roof scale model.

Laboratory

The CEL is a laboratory consisting of two chambers, thermally insulated, which contain a number of thermistors to monitor the temperature at all times, the CEL also has a control room where through computer equipment and specially designed software it is controlled and monitored the temperature of the thermistors in the two chambers. In this way the temperature inside the two chambers can be controlled and kept constant; the lower limit is 15 and 40°C is upper limit temperature.

Equipment and Materials

The experimental model is a wood parallelepiped with a square base. With dimensions of 20 cm at the base and total height of 35 cm; has no caps and has a zone division ata height of 15 cm. The walls are thermally isolated using glass wool and expanded polystyrene to ensure that heat transfer takes place preferably in the vertical direction.

In the lower zone there is a thermistor and at the top of the higher zone there is another thermistor; inside the higher zone there are 5 thermistors every 5 cm; this is illustrated in Fig. 3.

Experiment

The ice was placed in the lower zone of the parallelepiped with the purpose of having a cold constant temperature while the laboratory was programmed at a warm temperature. The parallelepiped is filled with soil to monitor the change in temperature at different heights and at different times, as is illustrated schematically in Fig. 3.

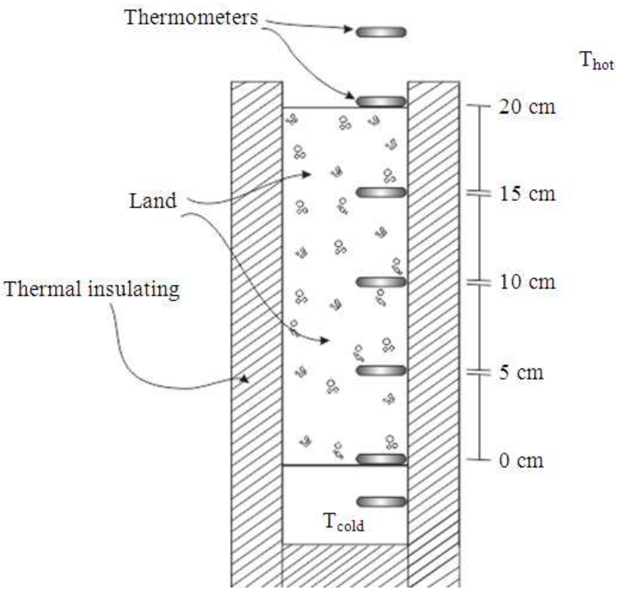

Fig. 3. Experimental model in Controlled Environment Laboratory (CEL)

During the experiment it was kept a constant cold temperature inside the experimental model and a hot temperature in the CEL in order to achieve a temperature gradient. Then the temperature was monitored on the soil until reaching the steady state.

Measurements

In order to determine if the mathematical model that includes expressions for the heat transfer through porous materials provides results comparable to those that would have in a green roof, the temperature of the soil as a function of distance and time in the presence of a temperature gradient was determined experimentally. Temperature at different distances is recorded every minute until reaching the steady state.

Comparisson Model

In this study the comparison of two experiments performed under different conditions of temperature is shown.

In the first case the laboratory temperature (warm temperature) was programmed at 35°C while the temperature inside the experimental model, i.e., the cold temperature was maintained around 12°C. These conditions were maintained about six hours and the results obtained are shown in Fig. 4.

Comparing the experimental data with the data obtained using the mathematical model simulation indicates that both curves show the same tendency.

Experimental data and simulation results obtained in the steady state are shown in Table 1. The maximum relative error obtained in this case is 13.81% at 15 cm distance, this may be because the soil properties are not uniform and change when approaching the interface.

In the second case the laboratory temperature (warm temperature) was programmed at 32°C while the temperature inside the experimental model, i.e., the cold temperature was maintained around 10°C. About six hours these conditions were maintained, the results obtained are shown in Fig. 5.

Experimental data and simulation results obtained in the steady state are shown in Table 2. The maximum relative error obtained in this case is 3.27% at 20 cm distance; as in the previous case, this may be because the soil properties are not uniform and change when approaching the interface, one of these properties is the volume fraction of each substance.

The results of simulation properly fit to the experimental data obtained in the laboratory. Therefore, the application of the Equation 12 and 15 within the heat transfer model for green roofs is suitable.

DOI: 10.3844/erjsp.2015.15.24

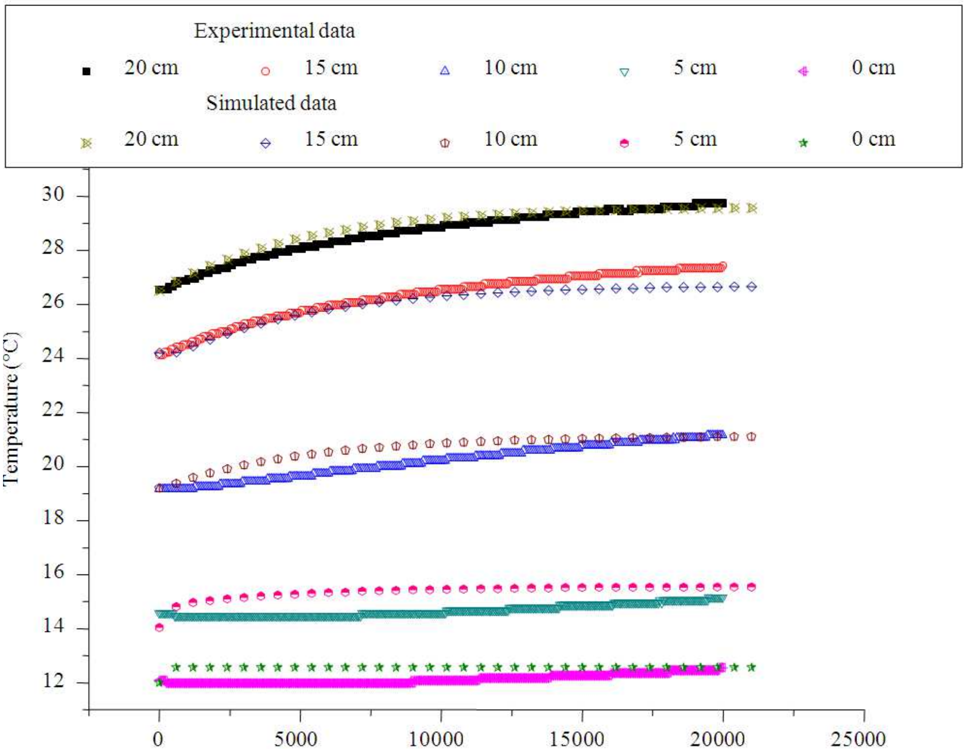

Fig. 4. Comparison of experimental data (lines) with simulated data (dots) at different heights (laboratory temperature 35°C)

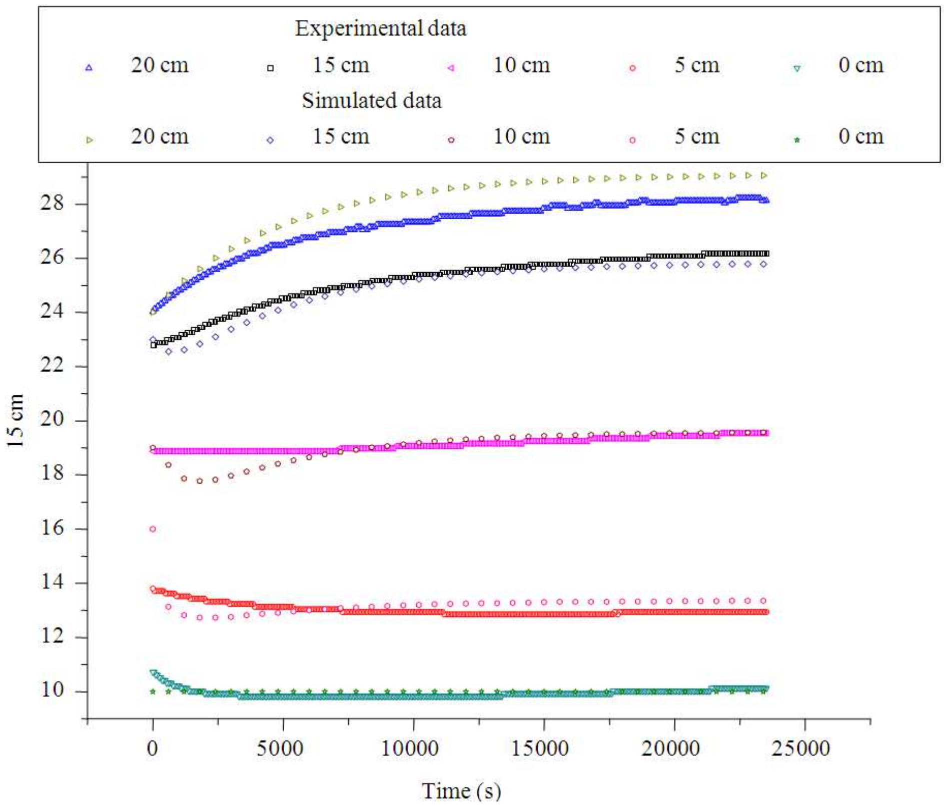

Fig. 5. Comparison of experimental data (lines) with simulated data (dots) at different heights (laboratory temperature 32°C)

Table 1. Comparing the experimental data with the results of simulation (first case)

	Distance (cm)				
Temperature	20	15	10	5	0
Experimental (°C)	29.730	27.4400	21.160	15.160	12.560
Simulation (°C)	29.560	23.6500	21.100	15.550	12.560
Relative error (%)	0.5718	13.8120	0.2836	2.5726	0.0000

Table 2. Comparing the experimental data with the results of simulation (second case)

	Distance (cm)				
Temperature	20	15	10	5	0
Experimental (°C)	28.130	26.170	19.540	12.940	10.120
Simulation (°C)	29.050	25.790	19.570	13.350	10.000
Relative error (%)	3.2705	1.4520	0.1535	2.1685	0.1858

Discussion

With the application of equations for porous materials it is possible to describe the heat transfer at the layers that make up green roofs. These equations have the advantage that they are easily applicable and contain the most important effects due to the properties of each material composing the layer under study.

Important effects on the canopy can not be taken into account if equations for porous media are not used. Among these effects we could have the shade provided by the canopy plus a change in the effective properties of the green layer.

Moreover it is clear that a higher density of plants in the green layer causes that the effective properties tend towards plant values, while properties for a lower density of plants will be more similar to those of air, similar behavior is also observed in the soil layer.

Likewise, through the right approach of boundary conditions it could be included important effects of heat sources in the soil layer.

Conclusion

In this study was proposed a new mathematical model to describe the heat transfer through green roofs. The main contribution is that the equations considering some layers of green roof consist of porous materials. The inclusion of porous media equations in the model was compared with experimental data. Good agreement between simulation and experiment exists for several experimental scenarios in a controlled environment laboratory.

Comparing the experimental data with the data obtained using the mathematical model simulation indicates that both curves show the same tendency.

On the other hand, a boundary condition is provided to integrate the heat sources involved in the model.

The mathematical model presented in this study can be used to compare the heat transfer through a green roof and a conventional roof in order to calculate the energy savings.

Apendice A

In this section it is presented the method used to move from the local-instantaneous formulation, for each of the phases in soil layer, to an equation valid for both phases. From the local instant formulation for earth and water vapor have:

$$\left(\rho Cp\right)_\ell \frac{\partial T_\ell}{dt} = k_\ell \nabla^2 T_\ell + q''' \quad (A.1)$$

$$\left(\rho Cp\right)_w \frac{\partial T_w}{dt} = k_w \nabla^2 T_w + q''' \quad (A.2)$$

The Equation A.1 and A.2 consider the heat transfer only by conduction, as the speed of the fluid phase is approximately zero.

On the other hand, average temperature of the land in the soil layer is given by Equation A.3:

$$\langle T_\ell \rangle = \frac{1}{V} \int_{V_\ell} T_\ell dV \quad (A.3)$$

where, V is the total volume and V_ℓ is the land volume. The average temperature of land on land is given by:

$$\langle T_\ell \rangle^\ell = \frac{1}{V_\ell} \int_{V_\ell} T_\ell dV \quad (A.4)$$

The Equation A.4 and A.5 are related by volume fraction:

$$\varepsilon_p = \frac{\frac{1}{V} \int_{V_\ell} T_\ell dV}{\frac{1}{V_\ell} \int_{V_\ell} T_\ell dV} \quad (A.5)$$

It averaging the left side of Equation A.1 and (A.6) is:

$$\left\langle \frac{\partial T_\ell}{dt} \right\rangle = \frac{1}{V} \int_V \frac{\partial T_\ell}{dt} dV \tag{A.6}$$

By applying Leibniz's theorem to Equation 6 we have Equation (A.7):

$$\left\langle \frac{\partial T_\ell}{\partial t} \right\rangle = \frac{\partial \langle T_\ell \rangle}{\partial t} + \frac{1}{V} \int_{A_{\ell w}} T_\ell w \cdot n_{\ell w} dA \tag{A.7}$$

where, $A_{\ell w}$ is the interfacial area between land and water, w is expansion volumetric coefficient for this case is zero and $n_{\ell w}$ is the unitary normal vector. Therefore the Equation A.7 can be written as:

$$\left\langle \frac{\partial T_\ell}{\partial t} \right\rangle = \frac{\partial \langle T_\ell \rangle}{\partial t} \tag{A.8}$$

Or:

$$\left\langle \frac{\partial T_\ell}{\partial t} \right\rangle = \varepsilon_\ell \frac{\partial \langle T_\ell \rangle^\ell}{\partial t} \tag{A.9}$$

Applying a similar treatment to the first term on the right side of Equation A1:

$$\nabla^2 T_\ell = \nabla \cdot (\nabla T_\ell) \tag{A.10}$$

$$\langle \nabla \cdot (\nabla T_\ell) \rangle = \frac{1}{V} \int_V \nabla \cdot (\nabla T_\ell) dV \tag{A.11}$$

$$\langle \nabla \cdot (\nabla T_\ell) \rangle = \nabla \cdot \langle \nabla T_\ell \rangle + \frac{1}{V} \int_{A_{\ell w}} (\nabla T_\ell) \cdot n_{\ell w} dA \tag{A.12}$$

$$\langle \nabla \cdot (\nabla T_\ell) \rangle = \nabla \cdot \left\{ \nabla \langle T_\ell \rangle + \frac{1}{V} \int_{A_{\ell w}} T_\ell \cdot n_{\ell w} dA \right\} + \frac{1}{V} \int_{A_{\ell w}} (\nabla T_\ell) \cdot \vec{n}_{\ell w} dA \tag{A.13}$$

$$\langle \nabla \cdot (\nabla T_\ell) \rangle = \nabla^2 \langle T_\ell \rangle + \nabla \cdot \left\{ \frac{1}{V} \int_{A_{\ell w}} T_\ell \cdot n_{\ell w} dA \right\} + \frac{1}{V} \int_{A_{\ell w}} (\nabla T_\ell) \cdot \vec{n}_{\ell w} dA \tag{A.14}$$

So you rewriting Equation A.1, we have Equation (A.15):

$$(\rho Cp)_\ell \varepsilon_\ell \frac{\partial \langle T_\ell \rangle^\ell}{\partial t} = k_\ell \left(\begin{array}{l} \nabla^2 \langle T_\ell \rangle + \nabla \cdot \left\{ \frac{1}{V} \int_{A_{\ell w}} T_\ell \cdot n_{\ell w} dA \right\} \\ + \frac{1}{V} \int_{A_{\ell w}} (\nabla T_\ell) \cdot \vec{n}_{\ell w} dA \end{array} \right) + \langle q''' \rangle \tag{A.15}$$

Defining an interfacial area density as:

$$a_{\ell w} = \frac{A_{\ell w}}{V} \tag{A.16}$$

Can rewrite the Equation A.15 as:

$$(\rho Cp)_\ell \varepsilon_\ell \frac{\partial \langle T_\ell \rangle^\ell}{\partial t} = k_\ell \varepsilon_\ell \nabla^2 \langle T_\ell \rangle^\ell + k_{\ell w} \varepsilon_w \nabla^2 \langle T_w \rangle^w + q_{\ell w} a_{\ell w} + \varepsilon_\ell \langle q''' \rangle^\ell \tag{A.16}$$

To apply the same procedure to Equation 2 we have:

$$(\rho Cp)_w \varepsilon_w \frac{\partial \langle T_w \rangle^w}{\partial t} = k_w \varepsilon_w \nabla^2 \langle T_W \rangle^W + k_{w\ell} \varepsilon_\ell \nabla^2 \langle T_\ell \rangle^\ell + q_{w\ell} a_{w\ell} \tag{A.17}$$

If is considering thermodynamic equilibrium, i.e., $\langle T_\ell \rangle^\ell = \langle T_w \rangle^w = \langle T \rangle$ and adding the Equation A.16 and A.17 we have:

$$(\rho_\ell Cp_\ell \varepsilon_\ell + \rho_w Cp_w \varepsilon_w) \frac{\partial \langle T \rangle}{\partial t} = (k_\ell \varepsilon_\ell + k_w \varepsilon_w) \nabla^2 \langle T \rangle + k_{\ell w} \varepsilon_\ell \nabla^2 \langle T_w \rangle^w + k_{w\ell} \varepsilon_w \nabla^2 \langle T_\ell \rangle^\ell + q_{\ell w} a_{\ell w} + q_{W\ell} a_{w\ell} + \varepsilon_\ell \langle q''' \rangle^\ell \tag{A.18}$$

Defining the following properties:

$$q_{\ell w} = \vartheta \left(\langle T_\ell \rangle^\ell - \langle T_\ell \rangle^\ell \right) \tag{A.19}$$

$$K_s = (k_\ell \varepsilon_\ell + k_w \varepsilon_w) \tag{A.20}$$

$$(\rho Cp)_s = (\rho_\ell Cp_\ell \varepsilon_\ell + \rho_w Cp_w \varepsilon_w) \tag{A.21}$$

Equation A.20 and A.21 are the effective properties of the layer of soil. Moreover it is considered that:

$$q_{\ell w} a_{\ell w} = -q_{w\ell} a_{w\ell} \quad \text{(A.22)}$$

Hence substituting Equation A.19-A.21 into Equation A.18 we have Equation (A.22):

$$(\rho Cp)_s \frac{\partial \langle T \rangle}{\partial t} = K_s \nabla^2 \langle T \rangle + \varepsilon_\ell \langle q''' \rangle^\ell \quad \text{(A.22)}$$

Acknowledgement

I would like to thank the CONACyT doctoral grant Dr. J.J. Ambriz-García for his support in the experiments at his research lab "Controlled Environment Laboratory (CEL)" of the Universidad Autónoma Metropolitana-Iztapalapa.

Funding Information

The research was funded thanks to the CONACyT doctoral grant to the first author.

Author's Contributions

The first author is completing his doctoral studies and the rest of the coauthors are part of his doctoral committee, in this sense all authors contributed in equal way in this publication.

Sergio Quezada-García: Performed all experiments, coordinated the data-analysis and contributed to the writing of the manuscript.

Manuela Azucena Escobedo-Izquierdo: Designed the research plan and organized the study.

Juan José Ambriz-García: Participated in all experiments.

Rodolfo Vázquez-Rodriguez: Participated coordinated the data-analysis and contributed to the writing of the manuscript.

Diego Morales-Ramírez: Designed the research plan and organized the study.

Ethics

We do not have any ethical issues.

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