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Analysis of Elastic-Plastic Contact Performance of Rigid Sphere Against a Deformable Flat-Effect of Strain Hardness

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Abstract: Problem statement: The present study considers an elastic-plastic contact analysis of a rigid sphere with a deformable flat (Rigid Sphere-model) using finite element analysis. The effect of tangent modulus on the contact behavior of a no adhesive frictionless elastic-plastic contact was analyzed using commercial finite element software ANSYS. **Approach:** Different materials, in terms of the ratio of Young's modulus to yield strength, had been considered to study the effect of tangent modulus. The Finite Element (FE) contact analysis was carried out by incorporating the various tangent modulus values of different materials. **Results:** The result clearly shows that for different tangent modulus the material hold different stress values. When this modulus increases the strain hardness value of material was also increased. **Conclusion:** With increase in tangent modulus, strain hardening resistance to deformation of a material is increased and the material becomes capable of carrying higher amount of load in a smaller contact area.

Key words: Tangent modulus, Young’s modulus, yield strength, E/Y ratio, elastic-plastic, strain hardness, rigid sphere

INTRODUCTION

The theory of contact mechanics concerned with the stresses and deformation which arise when the surfaces of two solid bodies are brought into contact. The two surfaces fit exactly or closely together without deformation (conforming contacts) and the surfaces, or one of the two surfaces, deforms when there is a contact area in between them (non-conforming contact). When two rough solids are brought to contact under a normal preload, contact junctions are formed at their contacting asperity tips, which may deform elastically, elastic plastic or plastic. The Stress and deflections arising from the contact between two solids have practical application in hardness testing, wear and impact damage of engineering ceramics, the design of dental prostheses, gear teeth and ball and roller bearings. In a non-conforming bodies, a contact area in between them is generally small when compared with the dimensions of the bodies themselves. The stresses are highly concentrated in the region close to the contact zone and they are not considerably influenced by the shape of the bodies at a distance from the contact area. The existing contact analysis is carryout based on the stress and strain in the contact bodies under loading and unloading conditions. The present study is to determine how the contact parameters are influenced in the load carrying capacity of the deformed body under loading condition an understanding tribological phenomenon such as contact fatigue, wear and damage.

Theoretical background: The metals hand book ASM Metals Hand Book defines hardness as “Resistance of metal to plastic deformation, usually by indentation. However, the term may also refer to stiffness or temper or to resistance to scratching, abrasion, or cutting. It is the property of a metal, which gives it ability to resist being permanently deformed when a load is applied. Another definition is that hardness measures the resistance to dislocation movement in the material, in

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which case it is directly related to the yield strength. A common definition that has gained status in the field is that hardness equals the average indentation pressure that occurs during fully plastic yielding of the contact area. Greater the hardness of the metal, higher is its resistantce to deformation. Here the hardness is defined "greater in the hardness of the metal, the greater resistant it has to deformation". The contact of a sphere and a deformable flat is a fundamental problem in contact mechanics with important scientific and technological aspects. The subject of normally loaded spherical contact stems from the classical study of Hertz in 1881 that derived an analytical solution for the frictionless (i.e., perfect slip) contact of two elastic spheres (Johnson, 1987). It is important to analyse either a single asperity contact or contacting rough surfaces consisting of multiple asperity contacts.

Two fundamental approaches have been studied for modeling a single asperity contact either considering a deformable hemisphere in contact with a rigid flat (Chang *et al*., 1987) (flatting approach) or by solving the contact mechanics problem of a rigid spherical indenter penetrating a deformable half space (Lin and Lin, 2006) (indentation approach). While in the elastic deformation regime, these two approaches are based on the Hertzian solution (Jackson and Green, 2005) and hence produce identical results. Whenever beyond the elastic deformation, these two approaches yield different contact mechanics response.

Literature review: Contact analysis can be traced back to 1882, in which Hertz studied the elastic contact between two glass lenses. Hertz theory is restricted to the normal frictionless contact between elastic half-space with small deformation. Abbott and Firestone (AF Model) (Abbott and Firestone, 1995) introduced the basic plastic contact model, known as the surface micro-geometry model. In this model, the deformation of a rough surface against a smooth rigid flat is assumed equivalent to the truncation of the undeformed rough surface at its intersection with the flat. Greenwood and Williamson (1966) used the Hertz theory and proposed an asperity based elastic model where asperity heights follow a Gaussian distribution. In order to bridge the two extreme models of GW (elastic model) and AF (plastic model), CEB model (Chang *et al*., 1987) developed an elastic-plastic contact model based on volume conservation of the plastically deformed asperities. Chang *et al*. (1988) introduced the hardness coefficient is related to the Poisson's ratio of the sphere. Kogut and Etison (2002) (KE model) used Finite element method solution for the elastic-plastic contact of a deformable sphere and a rigid flat by using constitutive laws appropriate to any mode of deformation, be it elastic or plastic. It also offers a general dimensionless solution not restricted to a specific material or geometry. Jackson and Green (2005) (JG model) incorporated variation of material property on deformed geometry and presented some empirical relations of contact area and contact load. Malayalamurthi and Marappan (2009) introduced a more accurate investigation of Finite Element (FE) analysis on the nature of material dependency of elastic-plastic contact behavior of deformable sphere and a rigid flat. Analysis is carried out between elastic limit till the inception of plasticity for various materials with different radii. The ratio of Young's modulus to yield strength (E/Y) value less than 300 show strikingly different contact phenomena. Shankar and Mayuram (2008) analyzed an axis-symmetrical hemispherical asperity in contact with a rigid flat is modeled for an elastic perfectly plastic material. This analysis shows the critical values in the dimensionless interference ratios ω/ω_c for the evolution of the elastic core and the plastic region within the asperity for different Y/E ratios. The FE Analysis of single asperity model with the elastic perfectly plastic assumption depends on the Y/E ratio of the material. Tabor (2000) proposed that hardness is not a unique material property. According to the literature review contact analysis of deformable sphere with a rigid flat using FE Analysis has done by several researchers and some of these studies consider the effect of material properties. The tangent modulus had been roughly considered as 10% of Young's modulus. Figure 1 shows that the RS-model (like as indentation approach). In the Brunel test a hard ball of diameter 'D' is pressed under a load 'W' into the plane surface under test.

After removal of the load, the choral diameter'd' of the resulting indentation is measured and the Brunel hardness HB is defined as the load W divided by the surface area of the spherical cap formed by the indentation Eq. 1:

$$| H_B = \frac{2W}{\pi D^2\left[1-\sqrt{1-(d/D)^2}\right]} \tag{1}$$

The Meyer hardness H_M, is determined by ball indentation in exactly the same way, but it is defined as the ratio between load applied and the projected area of the indentation, so that Eq. 2:

$$H_M = \frac{4W}{\pi d^2} \tag{2}$$

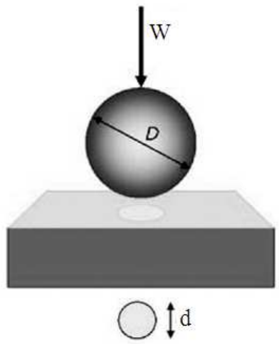

Fig. 1: Brinell hardness method

MATERIALS AND METHODS

The present study aims to study the effect of contact parameters such as contact area, tangent modulus and hardness for single asperity contact for different materials under loading condition of Rigid Sphere (RS) model. The Finite Element contact model of a rigid sphere against a deformable flat is shown in Fig. 2. The advantage of simulation, of axis-symmetric problems is that the spherical ball is considered as a quarter circles (Nakasone *et al*., 2006) For the RS-model contact analysis the contact pair is created between sphere and flat. The contact pair conformation is also shown in Fig. 2.

The meshed model is shown in Fig. 3. For this investigation ANSYS element type plane 82, conta172 and target 169 are used. The nodes lying on the axis of symmetry of the hemisphere are restricted to move in the radial direction. Also the nodes in the bottom of the hemisphere are restricted in the axial direction due to symmetry. The sphere size used for this analyses is R = 0.05 mt. Here frictionless rigid deformable contact analysis is performed for different materials. The material properties are given in Table 1.

Hardening parameter: In this analysis, a bilinear material property, is shown in Fig. 4. For linear hardening law 'H' is a constant and depends on the material parameters E and E_T:

$$H = \frac{E_T}{E - E_T} \quad (3)$$

Where:

E = Young's modulus value
E_T = Elastic Plastic tangent modulus,
Y_0 = Initial yield stress,
H = Hardness of the material

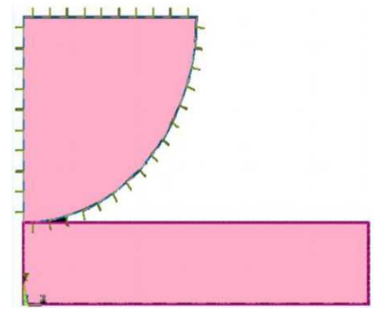

Fig. 2: Contact pair

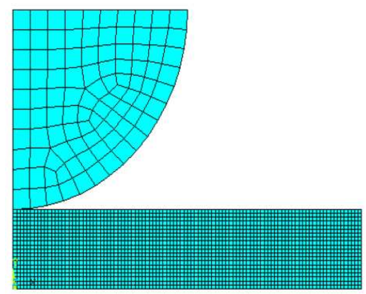

Fig. 3: Meshed model

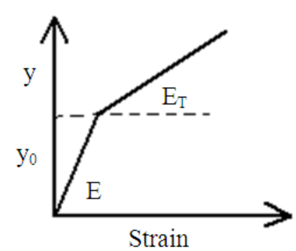

Fig. 4: Linear hardening law

Table 1: Material properties

E/Y	E×10^3 N/mm^2	Y×10^3 N/mm^2
552.63	210	380
736.84	70	95
769.23	100	130
1739.13	120	69

The tangent modulus is taken in terms of percentage of Young's modulus. It is found that the value of 'H' lies between 0 and 9 when using Eq. 3. It is obviously to check whether the obtained 'H' values will lies in the above limit leads to the validity of the new method. If H = 0 that indicates elastic perfectly plastic material (E_T = 0) behavior which is an idealized material behavior. The normalized general values of E_T and H is shown in Table 2.

Table 2: E_T and H values

Tangent modulus E_T	Hardness H
0	0.00
0.1E	0.11
0.2E	0.25
0.3E	0.43
0.4E	0.67
0.5E	1.00
0.6E	1.50
0.7E	2.33
0.8E	4.00
0.9E	9.00

Table 3: Tangent modulus and stresses

Tangent modulus	E/Y = 552.63 Stress (N/mm^2)	E/Y = 1739.13 Stress (N/mm^2)
0.1E	43.226	8.081
0.2E	43.066	7.815
0.3E	44.301	49.827
0.4E	56.812	21.976
0.5E	60.132	20.891
0.6E	59.932	22.477
0.7E	22.374	21.268
0.8E	15.679	8.264
0.9E	18.196	6.649

Table 4: Tangent modulus and stresses

Tangent modulus	E/Y = 736.84 Stress (N/mm^2)	E/Y = 769.23 Stress (N/mm^2)
0.1E	7.282	9.496
0.2E	2.990	11.712
0.3E	6.956	9.665
0.4E	16.563	15.347
0.5E	21.661	25.548
0.6E	6.664	24.102
0.7E	5.960	23.411
0.8E	5.992	20.902
0.9E	5.774	7.392

Table 5: Various contact parameters

E_T	ω	d	a/R	E^*a/ YR
0.1E	0.065	3.603	0.036	19.89
0.2E	0.062	3.519	0.035	19.34
0.3E	0.058	3.404	0.034	18.78
0.4E	0.055	3.315	0.033	18.24
0.5E	0.053	3.254	0.032	17.68

Penetration depth and projected surface: In this study an attempt has been made to modify the indentation depth in the new form by incorporating the tangent modulus in terms of %E. The loading relationship for the penetration depth is given by the relation:

$$\omega = \{9L^2/8D\}^{1/3}[\ 2\{(1-\nu^2)\ /\ E^* + E_T\ \}]^{2/3} \quad (4)$$

In Eq. 4, L is the applied load; D is the ball diameter and the paired material constants. ν, E^* and E_T are the Poisson's ratio, Equivalent young's modulus and tangent modulus. The E^* is given by:

$$1/E^* = \frac{1-\upsilon_1^{\ 2}}{E_1} + \frac{1-\upsilon_2^2}{E_2} \quad (5)$$

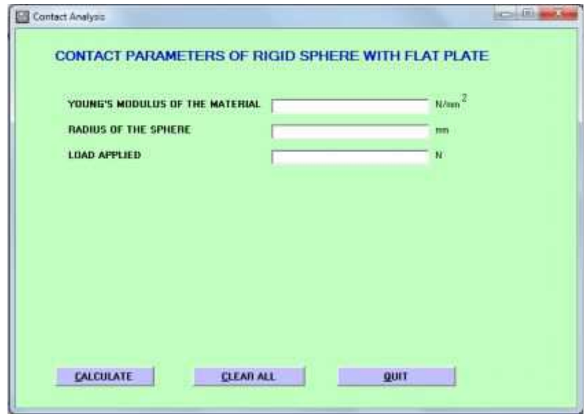

Fig. 5: Input view of design calculator

In Eq. 5, 1 and 2 denotes the ball and plate material properties respectively.

The projected surface diameter, d, of the residual impressed indentation was shown to fit the relationship:

$$d = 2\left[\omega\left(D - \omega\right)\right]^{1/2} \quad (6)$$

In Eq. 6, ω was taken as the total indentation depth under load and D is the ball diameter.

FE analysis: The wide range of values of tangent modulus is taken to make a fair idea about the effect of it in different materials, hardening parameter and the area of contact. The FE analysis is carried out for different materials i.e., 500≤ E/Y ≥1750. The stress values with respective to tangent modulus of different materials are given in Table 3 and 4.

Analytical solution for contact parameters: The indentation pressure under elastic, elastic-plastic and fully plastic conditions may be correlated using a non-dimensional form of p_m/Y as a function (E^* tan β/Y), where β is the angle of the indenter at the edge of the contact. With a spherical indenter tanβ ≈ sin β = a/R, which varies during indentation process. Where 'a' is width of the contact area (d/2) and 'R' is the radius of the ball (D/2). The material E/Y value of 552.63 is taken for observation of various parameters and it is related to the contact behavior of the sphere with flat (indentation approach) with the incorporation of the tangent modulus, Table 5 (Eq. 4-6).

Design calculator: The design calculator is developed using Visual-Basic coding (VB). Figure 5 shows the input model (Screen) of new developed calculator. The

Young's modulus of the material, radius of the sphere and load applied are the input data.

The VB coding is generated from the analytical elastic equations of the rigid sphere and a deformable flat, by considering the material properties.

RESULTS

Figure 6 shows the stress and Tangent modulus relationship. With the increase in tangent modulus value, the stress in the material (E/Y<1000) increases up to 0.5E. After that, stress decreases with increase in the tangent modulus. If stress in the material (E/Y > 1000) increases up to 0.3E, after that stress decreases with increase in the tangent modulus. Here it is observed that higher stress is developed in the material E/Y < 1000 of hardness H = 1 and H = 0.43 for the material having E/Y > 1000.

Figure 7 shows the diameter of projected surface of the residual impressed indentation. As the tangent modulus of the material increases this diameter(d) decreases.

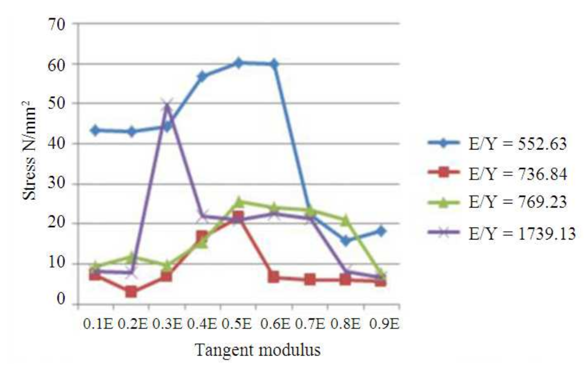

Fig. 6: Plot of stress Vs tangent modulus for different materials

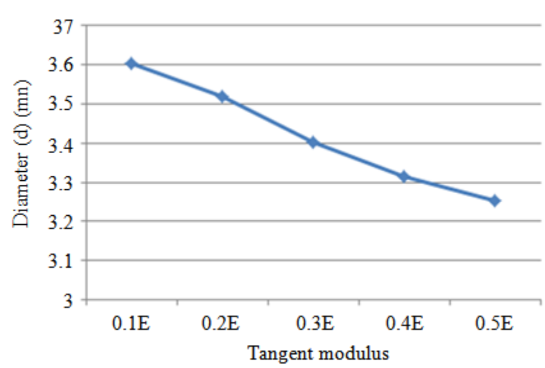

Fig. 7: Projected area diameter Vs E_T

The plastic strains are, of course, not uniform but, whatever their quantitative value, the strain will be a function of d/D. Then made a very bold assumption, namely that there is a representative strain, ε_T in the specimen which is a power function of d/D.

Figure 8 shows the relationship between the ratios of non-dimensional strain to tangent modulus. As the tangent modulus increases, the d/D ratio decreases. This is due to the projected surface diameter (d) decrease. This implise that the tangent modulus increase.

Figure 9 shows the output model of new design calculator. The output parameters are radius of contact, contact pressure, area of contact. Maximum stress induced and depth of penetration.

The New design calculator is developed by computer coding for calculating the various contact parameters. It is very useful for basic learner and design engineer for the selection of material for designing a component.

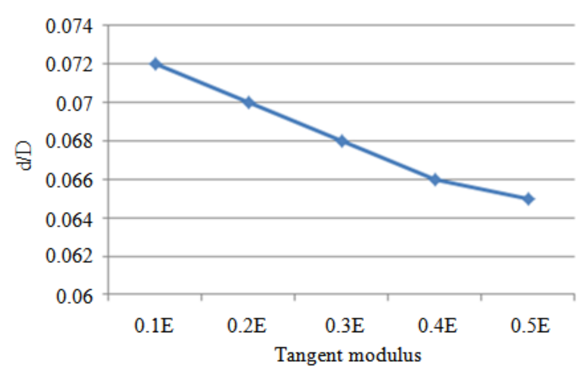

Fig. 8: d/D ratio Vs E_T

Fig. 9: Output view of design calculator

DISCUSSION

From the results obtained it is observed that the non-linear behavior in-between stress and tangent modulus. The tangent modulus increases the hardness of the material. The material behavior is dependent on the tangent modulus. The effect of tangent modulus has greater influence in contact parameter.

CONCLUSION

The tangent modulus of the material is not considered in the study of rigid sphere and a deformable flat model so-far. The effect of the tangent modulus in the contact parameters is very important for contact phenomena. The detail study of the effect of tangent modulus and strain hardness is carried out by FE analysis and analytical solutions. The different materials were consider for the analysis. In the end it was found out that the stress hold in the material is depends upon the tangent modulus value of the material. It is observed that the higher stress is developed in the material $E/Y < 1000$ of hardness $H = 1$ and $H = 0.43$ for the material having $E/Y > 1000$. It is established that when the tangent modulus is increased, the hardness of the material too increases. The increase in tangent modulus also reduces the projected area of the indentation. So the d/D ratio is decreased when the tangent modulus increases. The reduction in this ratio implies the increase of the straining action of the material. The material can cary large load in smaller contact area when the straining action (Strain hardening) is increased. VB coding was generated to develop a design calculator for calculating the various contact parameters. It is very useful for design engineers to select the suitable material based on the material properties for designing a component under loading contact condition.

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